The Duality Horizon: Extra-Spatiotemporal Structure as the Minimal Completion of Physical Reality

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ABSTRACT

Three of the most precisely confirmed results in modern physics—loophole-free Bell non-locality, quantum measurement-outcome selection under the Born rule, and holographic entropy bounds establishing non-volumetric information scaling—are standardly treated as independent foundational anomalies. This paper argues that they are not independent: they are three observational projections of a single structural absence in Physical Matter Reality (PMR). Each requires a global consistency constraint acting on PMR from outside its own dynamical rule set; each is traceable to the same formal closure failure; and three major research programmes—the geometric interpretation of entanglement, the information-theoretic resolution of the measurement problem, and the holographic emergence of spacetime—are independently converging on the same unnamed substrate that supplies what PMR cannot supply for itself.

We introduce Non-Physical Matter Reality (NPMR) as the minimal extra-spatiotemporal informational domain that simultaneously restores closure under all three constraints. Its influence is encoded in a representational map $f: X_{\mathrm{N}} \to \mathrm{Prob}(X_{\mathrm{P}})$ and quantified by a structural coupling κ_{NPMR}, measured through three observational windows whose joint non-vanishing is the framework's signature prediction. We argue that both PMR and NPMR must be finite-dimensional and — under explicitly stated structural assumptions: a Gradient Dominance Hypothesis, a Degeneracy Conjecture, and two representational postulates — derive the Duality Horizon Ω as the subset of the joint state space at which the representational map degenerates. The Reset Theorem then establishes, conditionally on these assumptions, that the unique consistent continuation of any trajectory approaching Ω is a global reset to the maximal-entropy initialisation state. Cosmological cyclicity, the thermodynamic arrow of time, and the absence of infinite ontological regress follow as corollaries of this conditional structure.

The paper does not seek direct empirical confirmation of NPMR but inverts the burden: NPMR is the minimal structure consistent with all established empirical constraints; it stands until a more parsimonious alternative is produced. Three classes of falsifiable structural predictions are identified and four challenges issued to any competing framework. The epistemic status of every claim — proved, conditional, or conjectural — is flagged explicitly throughout.

Keywords: quantum foundations · measurement problem · Bell non-locality · holographic principle · ontology of spacetime · cyclic cosmology

1 Three Roads to the Same Horizon

There is a particular kind of theoretical blindspot that emerges not from ignorance but from specialisation. Three research programmes, each rigorous and well-funded and internally coherent, can spend decades converging on the same structural gap without any of them recognising the convergence—because each is facing inward toward its own formalism rather than outward toward the common horizon they are all approaching.

This paper is written from that horizon.

The programmes in question are the geometric interpretation of quantum entanglement, the information-theoretic resolution of the measurement problem, and the holographic emergence of spacetime. The gap they are converging on is the same in each case: a substrate that is real, causally efficacious, and irrecoverably outside the spatiotemporal framework within which all three programmes operate. Each field has named its own version of this substrate. None has recognised that the others are naming the same thing. And none, as a consequence, has yet attempted a unified formal account of what that substrate actually is.

That is what this paper provides.

1.1 The Instrument You Cannot See

We will develop a single analogy throughout this paper, adding precision at each stage as the argument deepens. (→ C) It begins simply.

Imagine an instrument of cosmological scale—its strings so taut, its vibrational frequency so far beyond any detectable spectrum, that from inside the spacetime world it produces the instrument is completely invisible. You can hear the notes. You can measure their correlations with arbitrary precision. You can confirm, beyond any experimental doubt, that no signal passed between the striking of a key and the sounding of a note at the far end of the instrument. And yet the correlation is perfect.

The naive response is to hunt for the hidden signal. The correct response is to recognise that you are looking at a piano and asking why the strings move together, without knowing there is a piano. The string does not transmit. It is the connection. The architecture predates the note. And the architecture—the frame, the bridges, the resonant body—is not made of sound. It operates at a level beneath what sound can describe.

Now make one further modification. Suppose the instrument is built from a material so dense, and its strings so taut, that the vibrational frequency lies entirely beyond the range of human perception—not merely outside our hearing, but outside the entire electromagnetic spectrum we use to observe the world. We cannot see the strings. We cannot feel the frame. We cannot detect the medium through which the vibration propagates because that medium is not in our spacetime. It is the structure beneath it.

From inside such a world, you would observe only the notes. You would hear sounds appearing at correlated moments with no detectable mechanism of transmission. You would measure the correlations, confirm with extraordinary precision that no signal could have passed between the events, and be left with a profound and apparently insoluble mystery.

Unless someone told you about the piano.

1.2 The First Road: Entanglement as Geometry

The Bell results [9, 35, 40, 65] established a hard negative: no local realist account survives contact with experiment. But what has happened since is more interesting than a prohibition. The question has shifted from what entanglement rules out to what it rules in—and the answer, developed with increasing formality since Maldacena and Susskind's ER=EPR conjecture [45], is that entanglement rules in geometry.

The conjecture has hardened considerably. Engelhardt and Liu's 2024 algebraic formulation [28] establishes that the operator algebra of entangled subsystems carries precisely the structure of a connected geometry—the entanglement entropy between regions directly encoding the area of the minimal surface connecting them in the dual gravitational description. Work on strongly-coupled quantum matter in 2025 [66] describes the wormhole not as a tunnel through which something travels but as "a mechanism for long-range interaction without causal suppression outside the lightcone"—a standing structural connection beneath the causal order of spacetime, not subject to it.

Van Raamsdonk's formulation [75] remains the most direct: disentangle two regions and they pull apart. Spacetime connectivity is not the container within which entanglement lives. Entanglement is the substrate from which spacetime connectivity is built.

This programme has therefore arrived, under its own theoretical pressure, at a structure that predates the geometry it generates—a structure that is neither spatial nor temporal, but from which both properties emerge as consequences. It has arrived at one face of the horizon. It does not yet have a name for what it found there.

1.3 The Second Road: What Selects the Outcome

Quantum Darwinism represents the most significant advance on the measurement problem since decoherence, and its experimental confirmation in 2025 using superconducting circuits [88] is genuine progress. The redundant broadcasting of pointer-state information through environmental fragments [15, 91]—the mechanism by which multiple observers converge on the same classical fact without having disturbed the system—is a real and now experimentally established phenomenon. The question of why observers agree has an answer.

But the question of what selects the specific outcome remains exactly where it was. Decoherence explains the disappearance of interference terms. Quantum Darwinism explains the consensus among observers. Neither explains why a single value obtains in a given run rather than another, nor why the distribution of obtained values across many runs obeys the Born rule globally—that is, consistently across space-like separated measurement events involving entangled systems [90].

This consistency is not a local fact. It cannot be enforced by any mechanism confined to the neighbourhood of either measurement. It is a global constraint on the joint state space—one that acts instantaneously across arbitrary separations, respects no-signalling, and leaves no dynamical trace in the local evolution of either system. It is, in other words, a constraint that operates from outside the local causal structure entirely.

The measurement programme has therefore arrived, from a completely different direction, at the same structural gap: something acts on PMR from outside its own dynamical rules, imposes globally consistent selections, and leaves no directly observable signature beyond the statistics of outcomes. This is the second face of the horizon.

1.4 The Third Road: The Boundary Knows More Than the Bulk

The holographic principle has been established long enough that its most radical implication is in danger of becoming furniture—something physicists cite without quite feeling its full weight. The implication is this: the volumetric description of space is wrong. Not approximately wrong. Categorically wrong. The maximum information content of a three-dimensional region scales with its two-dimensional boundary [8, 70, 72], which means the third dimension is, in a precise information-theoretic sense, redundant. The bulk is derived. The boundary is primary.

The recent developments push this further than Bekenstein and Hawking could have anticipated. The island formula [3, 50], resolving the black hole information paradox, demonstrates that information encoded in the interior of a black hole is recoverable from a distant boundary—that interior and exterior are not informationally independent in the way a volumetric description would require. And Takayanagi's 2025 work on pseudo-entropy and time-like entanglement [71, 94] now proposes that the temporal coordinate is itself emergent—encoded in the imaginary component of boundary entropy, arising from the entanglement structure of the boundary rather than being a fundamental feature of the system.

Not just space. Time. Both classical dimensions of spacetime are increasingly understood as derived quantities, encoded on a boundary that is itself neither spatial nor temporal in the ordinary sense, and that carries strictly more information than the spacetime it generates.

The holographic programme has therefore arrived at the third face of the same horizon: a boundary structure that is ontologically prior to spacetime, that encodes spacetime's geometry and dynamics, and that cannot be described in the vocabulary of the physics it produces.

1.5 What Is Waiting at the Intersection

Three programmes. Three independent trajectories, each driven by different empirical results and theoretical pressures. Each converging on a structure with the same four properties: it is real and causally efficacious; it operates outside the spatiotemporal causal order; it enforces global consistency constraints on locally evolving systems; and it is inaccessible to direct observation from within the physics it constrains.

Programme	Name for the gap
Entanglement research	The substrate of geometry
Measurement theory	The global selection constraint
Holographic physics	The boundary from which spacetime emerges

These are not different structures. They are three partial descriptions of one structure—seen from three different directions by three communities that have not yet compared notes.

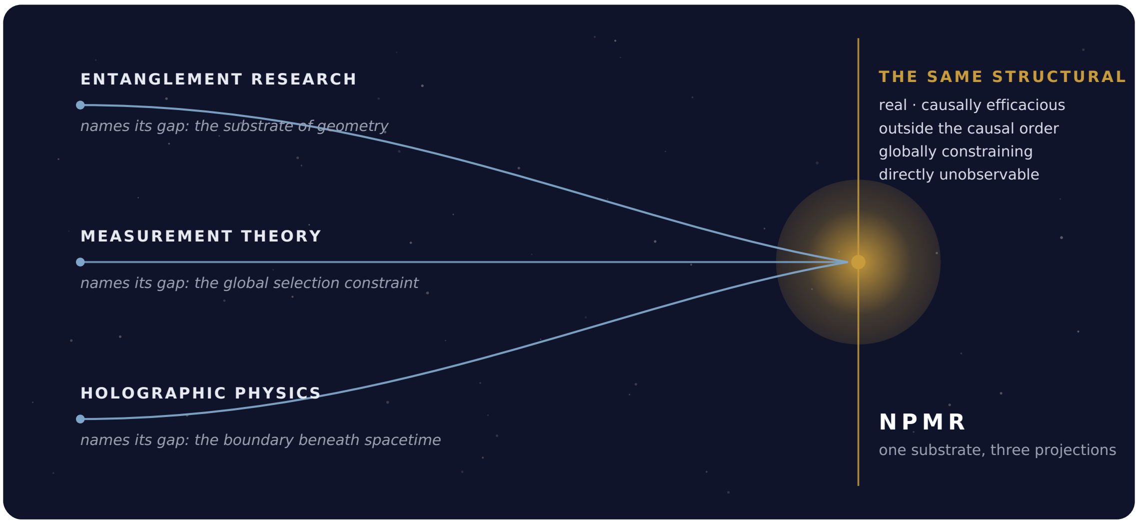

Figure 1 — Three research programmes, one structural gap. Entanglement geometry, outcome selection, and holographic bounds each terminate at the same extra-spatiotemporal substrate.

The structure has no agreed name. This paper gives it one.

Non-Physical Matter Reality—NPMR—is introduced here as a postulate, but a motivated one: the formal designation for what the convergence of three independent programmes makes the parsimonious completion of PMR. The convergence of three independent research programmes on the same structural gap is not a coincidence to be explained away. It is, as we will argue in the sections that follow, the signature of a genuine ontological boundary—the point at which physics conducted from inside spacetime encounters the limits of what spacetime-bound inquiry can resolve. (→ B.5)

A note on terminology. The paired terms Physical Matter Reality and Non-Physical Matter Reality, with these acronyms, originate in Thomas Campbell's *My Big TOE* [93], where they name a consciousness-centred metaphysics in which PMR is rendered within a larger conscious system. The present paper adopts the vocabulary for its descriptive precision — an extra-spatiotemporal domain deserves a name that says so — while departing from Campbell's ontology at every load-bearing point: NPMR is here a formally constrained informational domain, derived as the minimal closure-restoring structure for three empirical constraint classes, and no role is assigned to consciousness, observers, or simulation (→ 3.7). The debt is terminological, and it is acknowledged as such.

1.6 The Epistemological Contract

We are not seeking empirical confirmation of NPMR in the direct sense. We cannot—not because the theory is weak, but because of what NPMR is. A domain that operates outside the spacetime framework will leave only indirect, structural signatures in spacetime-bound measurements. Those signatures are precisely what entanglement research, measurement theory, and holography have been cataloguing. The confirmation is already present. What has been absent is the unified account.

The methodology here is falsificationist burden-inversion. NPMR is the minimal structure simultaneously capable of resolving the open residuals of all three programmes. It stands until a more parsimonious alternative is produced. The argument is not here is evidence for NPMR—it is here is what the evidence already implies, and here is a formal proof that nothing less will do.

The universe does not make unnecessary detours. When three independent roads arrive at the same gap, the most parsimonious inference is that the gap is real, and that what lies beyond it is a single thing.

1.7 The Piano, Continued

We can now hear the instrument more precisely.

The entanglement programme has been measuring the tension in the strings—finding that what appears as correlation across space is in fact the behaviour of a connected structure beneath it, one that is not subject to the causal constraints of the spacetime within which the correlated events appear to be separated.

The measurement programme has been studying why only certain notes become audible—finding that classical reality is the redundantly broadcast signal of environmentally selected states, but discovering that the selection rule itself is not in the local dynamics. It is imposed from a level the local dynamics cannot reach.

The holographic programme has been examining the resonant body of the instrument—finding that the apparent volume of space is encoded on its surface, and now that even the direction of time is a feature of boundary structure rather than a fundamental property of the bulk.

Each programme has been studying a different component of the same instrument. The piano is real. Its frame predates the spacetime it produces. Its strings connect what appears to be separate. Its resonant body is not where the notes suggest it should be.

NPMR is the name of the instrument.

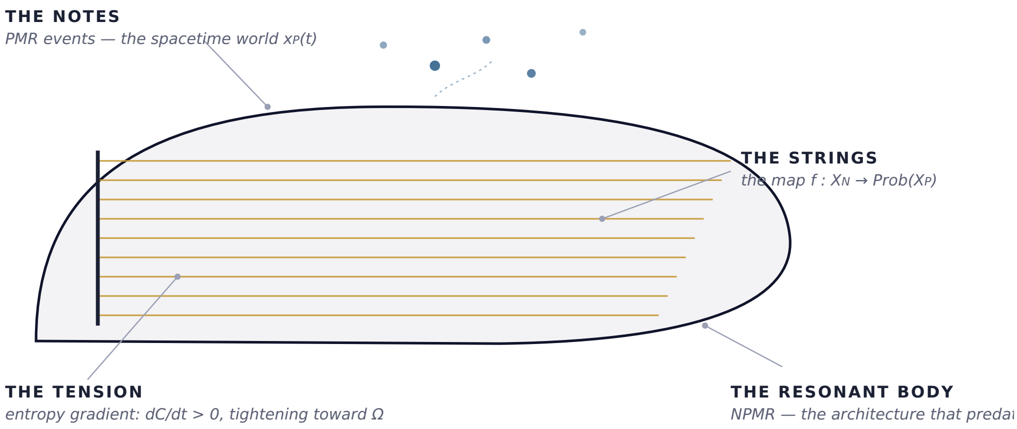

Figure 2 — The instrument. The piano analogy mapped to its formal counterparts: strings as the representational map, resonant body as NPMR, notes as PMR.

What follows is its formal description—beginning with the precise sense in which the physics we can observe is insufficient to account for itself, and ending with the boundary condition that every finite informational system must eventually reach. (→ C)

2

The Formal Closure Problem: Why PMR Cannot Account for

Its Own Phenomena

Physics has always advanced by discovering that the world contains more than the current framework can hold. Newton's mechanics could not account for electromagnetism; Maxwell's equations could not account for the constancy of the speed of light; special relativity could not account for gravity. In each case, the response was not to abandon the existing framework but to embed it within a larger one—one that preserved everything the original theory correctly described while adding the structure necessary to account for what it could not.

We are proposing that physics is at such a moment now. The framework that cannot hold the world is not classical mechanics, or special relativity, or even quantum field theory taken in isolation. It is the assumption, shared across all of these frameworks, that the physical universe is a closed system—that everything that happens within it can, at least in principle, be accounted for by rules that operate entirely within it.

This assumption has a name in the philosophy of physics: ontological closure. And it is, we will argue, demonstrably false. Not approximately false, not false in some interpretive or philosophical sense, but formally and empirically false—falsified by three independent classes of phenomena that are among the most precisely confirmed results in the history of science.

The argument of this section is structural. We will define closure precisely, show what its failure means formally, and then demonstrate that the three research programmes described in Section 1 are each, at their core, a different manifestation of the same closure failure. (→ A.1, A.2)

2.1 What Closure Means, and Why It Matters

Model the physical universe—everything in spacetime, everything subject to physical law, everything in principle observable—as a dynamical system. Following the information-theoretic tradition running from Wheeler's "it from bit" programme [80] through Zeilinger [87] to the contemporary quantum gravity literature, we write this system as:

$$S = (X, R)$$

where X is the set of all physically realizable states—quantum states, classical configurations, measurement records, spacetime geometries, correlations—and R is the complete rule set governing how states evolve: Schrödinger dynamics, Einstein field equations, decoherence channels, gauge interactions. We call this system Physical Matter Reality, or PMR. (→ A.1)

> **Definition 2.1** (Ontological closure). PMR is ontologically closed under its rule set R if and only if

$$X = \mathrm{cl}(R), \qquad \text{where} \qquad \mathrm{cl}(R) := \{\, x \in X \mid x \text{ is generable by } R \,\}.$$

This is stronger than asking whether PMR is predictable, or computable, or epistemically accessible. Those are questions about what we can calculate or know. Ontological closure is a question about what exists and what generates it. It asks whether the rules of physics, taken in their entirety, are sufficient to produce everything that physics describes—or whether some phenomena in X require something outside R to bring them about.

The distinction matters because the history of physics offers a clear pattern: whenever a formal system is found to violate closure, the correct response is not to patch the system but to embed it within a larger one that supplies the missing structure. The rationals violate closure under limits—the response is the real numbers. The reals violate closure under algebraic inversion—the response is the complex numbers. Classical phase space violates closure under quantum measurement—the response is Hilbert space. Local realist models violate closure under Bell correlations—the response is, as we are about to argue, the structure this paper describes. (→ A.2, B.4)

In each case, the closure-restoring structure is not optional. It is the minimal domain within which the original system can be consistently embedded. The question is never whether to extend the ontology. The question is how to extend it with minimum additional machinery.

2.2 The First Failure: Non-locality and the Limits of Local Rules

Bell's 1964 theorem [9] established that no theory satisfying locality and realism can reproduce all the predictions of quantum mechanics. For two decades this remained a theoretical result. The experiments of Aspect, Grangier, and Roger in 1982 [5] provided compelling confirmation. And in 2015, three independent groups—Hensen et al. in Delft [40], Giustina et al. in Vienna [35], and Shalm et al. in Boulder [65]—simultaneously closed the remaining loopholes, producing loophole-free Bell violations at statistical confidences that render every local realist alternative not merely implausible but formally excluded. (→ B.1, B.5)

The formal structure of the result is worth stating precisely, because it is easy to understate its significance. Two space-like-separated observers, Alice and Bob, measure entangled particles with freely chosen settings a and b, obtaining outcomes A, B $\in \{-1, +1\}$. Local realism requires outcomes to depend only on local settings and shared hidden variables λ: A = A(a, λ), B = B(b, λ). This yields the CHSH inequality [20]:

$$|S_{\text{CHSH}}| \leq 2, \qquad S_{\text{CHSH}} = E(a_0, b_0) + E(a_0, b_1) + E(a_1, b_0) - E(a_1, b_1).$$

Quantum mechanics predicts $|S_{\text{CHSH}}{}^{\text{QM}}| = 2\sqrt{2} \approx 2.828$. Loophole-free experiments confirm $|S_{\text{CHSH}}{}^{\text{exp}}| > 2$ with $p \ll 10^{-7}$. Therefore:

$$C_{\text{exp}} \notin \text{cl}(R_{\text{local}}).$$

Local realist rule sets are formally falsified. (→ A.2)

If PMR's rule set R is local—if rules propagate influence only within the spacetime light cone—then the correlations that appear in X cannot be elements of $\text{cl}(R_{\text{local}})$. PMR either violates locality (incompatible with relativistic causal structure), adopts superdeterminism (incompatible with scientific inference), or is not ontologically closed. The third option is the only one compatible with both experimental data and scientific methodology [10]. (→ A.2, D)

What the Bell results establish is not merely that quantum mechanics is strange. They establish that the physical universe contains correlations that its own local rules cannot generate. Something outside the local dynamical structure enforces these correlations. The piano string connects the keys before any note is struck. (→ B.1)

2.3 The Second Failure: The Outcome Selection Gap

The measurement problem is the oldest unresolved problem in quantum foundations, and it has resisted resolution for nearly a century not because physicists have lacked ingenuity but because the problem is, at its root, a closure failure—and closure failures cannot be resolved by working harder within the closed system.

Quantum mechanics describes two dynamical modes. The first is unitary evolution under the Schrödinger equation:

$$|\psi(t)\rangle = e^{-iHt/\hbar}\,|\psi(0)\rangle.$$

This is linear, reversible, and deterministic. It takes superpositions to superpositions. The second mode is outcome selection: when a measurement is performed on a system in state $|\psi\rangle = \Sigma_i\, \alpha_i |i\rangle$, a single definite outcome o_i is obtained with probability $|\alpha_i|^2$. This is non-linear, irreversible, and stochastic.

The problem is that the first mode cannot produce the second. No composition of linear, reversible, deterministic operations yields a non-linear, irreversible, stochastic one. Decoherence—the process by which environmental interaction suppresses interference terms in the reduced density matrix—explains why superpositions become locally invisible. It does not explain why one outcome obtains rather than another, nor why the distribution of outcomes across many runs is globally consistent with the Born rule across space-like separated measurements [63, 90]. (→ B.2)

Quantum Darwinism, as described in Section 1, resolves the question of intersubjective agreement—of why multiple observers converge on the same classical fact [15, 88, 91]. It does not resolve the question of outcome selection itself. As Zurek acknowledges explicitly [92], the randomness of quantum jumps—the fact that a specific outcome obtains in a specific run—remains unexplained by decoherence or Darwinism alone.

The formal statement of the closure failure is clean:

$$o_i \notin \mathrm{cl}(R_{\mathrm{unitary}}).$$

Definite outcomes are elements of X—they are unambiguously part of physical reality, the most immediately present facts of experimental experience. They are not elements of the closure of unitary dynamics. PMR's primary dynamical rule set cannot generate the most basic facts that PMR contains. (→ A.2)

There is a further constraint that sharpens the failure. Outcome selection is not merely locally underdetermined—it is globally constrained. When Alice and Bob measure entangled particles at space-like separation, the outcomes they obtain must be mutually consistent with the quantum correlations predicted by the shared state. This consistency is enforced across an arbitrary spatial separation, instantaneously, in a way that respects no-signalling. No local mechanism—no hidden variable, no environmental interaction, no decoherence channel—can enforce a global consistency constraint of this kind. The constraint acts on the entire joint state space simultaneously. It comes, necessarily, from outside the local causal structure. (→ A.2, A.3)

In piano terms: it is not merely that the note selection is mysterious. It is that the consistency of simultaneously struck notes across the full length of the instrument—their harmonic coherence—cannot be explained by any mechanism local to either key. The harmonic structure is a property of the instrument, not of the individual keys.

2.4 The Third Failure: The Representational Deficit

The third closure failure is the most architectural of the three. It concerns not specific phenomena within PMR but PMR's capacity to represent itself—its ability to encode, within its own degrees of freedom, the full information content of its own states.

Bekenstein's 1973 discovery [8] that black hole entropy scales with horizon area rather than enclosed volume, and Hawking's confirmation of the associated temperature [38], established that the thermodynamic properties of gravity are fundamentally two-dimensional in character. 't Hooft [72] and Susskind [70] generalised this into the holographic principle: the maximum information content of any spatial region is bounded by the area of its bounding surface, not its volume. (→ B.3)

The significance for PMR closure is precise. A volumetric description of space assigns degrees of freedom proportional to volume. The holographic bound assigns degrees of freedom proportional to area. The volumetric description is therefore not merely an approximation—it is a systematic overcount of the true degrees of freedom available to physical reality. PMR, treated as a three-dimensional field theory, contains more apparent degrees of freedom than it is physically permitted to have. Write ${I_{\mathrm{bulk}}}^{\mathrm{naive}}$ for this apparent volumetric count, $I_{\text{bulk}}^{\text{dyn}}$ for the information actually generable by PMR-local dynamics, and I_{boundary} for the holographic capacity.

This overcount cannot be corrected internally. The constraint that enforces area-scaling acts globally—it eliminates bulk microstates by reference to a boundary that is not itself a feature of the bulk. The boundary carries information that the bulk cannot generate by its own dynamics. (→ A.2, A.4)

The island formula, developed by Penington [50] and Almheiri, Engelhardt, Marolf, and Maxfield [3], makes this concrete. Information apparently swallowed by a black hole and rendered inaccessible to bulk observers is in fact encoded on a distant boundary surface—recoverable in principle, consistent with unitarity, but not reconstructible from any purely local bulk description. The bulk is informationally incomplete. The boundary knows more than the bulk can see. (→ B.3, B.5)

And Takayanagi's 2025 work on pseudo-entropy and time-like entanglement [71, 94] now proposes that the temporal coordinate itself is encoded in boundary structure—that the emergence of the time direction is a holographic phenomenon, arising from what he calls time-like entanglement on the boundary. PMR does not merely fail to encode all of its own spatial information. It fails to encode its own temporal structure.

The closure failure is then two-sided — an overcount that only an external constraint removes, and a deficit that no internal rule can fill:

$$I_{\text{bulk}}^{\text{naive}} > I_{\text{boundary}} > I_{\text{bulk}}^{\text{dyn}}.$$

PMR cannot represent itself. The instrument's resonant body is not inside the space the instrument produces. (→ A.4)

2.5 One Failure, Three Faces

We have now described three closure failures. It is important to see that they are not independent.

Bell non-locality establishes that PMR contains correlations its local rules cannot generate—that the coordination of distant events exceeds what local dynamics permits.

Outcome selection establishes that PMR contains definite facts its unitary dynamics cannot produce—that the selection of specific events from quantum probability distributions requires a constraint external to those dynamics.

Holographic bounds establish that PMR cannot encode its own information content—that the representation of physical reality requires degrees of freedom that do not reside within the bulk spacetime.

In each case, the same structure is implicated: something that acts on PMR globally, enforces consistency constraints across arbitrary separations, operates outside the spacetime causal order, and leaves no directly observable signature beyond the structural features of PMR that it constrains. The Bell constraint coordinates. The outcome-selection constraint chooses. The holographic constraint encodes. But the entity doing the coordinating, choosing, and encoding is, in all three cases, the same kind of thing—and in all three cases, it is the same kind of thing that cannot be inside PMR.

This is the unified closure failure. PMR is not broken in three places. It has one structural absence that appears as three symptoms—the way a piano without its resonant body would produce strings that vibrate without projecting sound, keys that respond without coordinating harmonically, and a structure that appears to contain more potential notes than it can actually produce.

> **Proposition 1.1 (Unified Closure Failure).** PMR, modelled as the dynamical system

S = (X, R), fails ontological closure under three independent and empirically confirmed constraints: Bell non-locality, Born-rule outcome selection, and holographic information bounds. These three failures are structurally unified: each requires a global consistency constraint acting on PMR from outside its own dynamical rule set. No mechanism confined to the spacetime manifold can supply this constraint without violating Lorentz invariance, the no-signalling theorem, or the free-setting independence required for valid experimental inference. (→ A.2, A.3)

The minimal structure capable of supplying what PMR cannot supply for itself—and the formal proof that nothing less will do—is the subject of Section 3.

3

NPMR: The Minimal Postulate

Every significant extension in the history of physics has followed the same logical sequence. First, a framework is found to be incomplete—not merely approximately wrong at the edges, but structurally insufficient at the centre. Then, the minimal extension capable of restoring completeness is identified. Then, and only then, that extension is given a name and its properties are investigated.

> **The postulate, stated plainly.** The argument of this section rests on a claim it does not pretend to prove from within physics: that the three closure failures of Section 2 are one failure, and that no account internal to PMR can close it. NPMR is the minimal structure that follows once this is granted. The convergence of the three programmes is the *motivation* for the postulate, not a derivation of it — and the whole framework is offered in the burden-inverting spirit of §6: it stands until the convergence is shown to be illusory, or a more parsimonious completion is produced. What follows should be read as the consequences of this postulate, rigorously drawn, rather than as a proof that reality must contain NPMR.

The real numbers were not proposed and then found to resolve the incompleteness of the rationals. The incompleteness was found first, and the reals were derived as the minimal closure-restoring structure. The same is true of complex numbers, of Hilbert space, of general relativity as an extension of special relativity. In each case, the extension was not a speculative addition to a working theory. It was a structural necessity derived from the theory's own failure to close.

We proceed in exactly this tradition. Section 2 established the failure. This section derives the extension.

The argument has three movements. First, we identify the properties that any closure-restoring structure must possess—not by assumption, but by reading them directly off the three faces of the closure failure. Second, we prove that no structure possessing these properties can reside within the spacetime manifold. Third, we formally define NPMR as the minimal extra-spatiotemporal domain satisfying all required properties, introduce the structural constant κ_{NPMR}, and confront directly the question of what NPMR positively is. (→ A.1, A.2, A.3)

3.1 Reading the Requirements Off the Failures

Proposition 1.1 established that PMR requires a global consistency constraint acting from outside its own dynamical rule set. We can now be precise about what that constraint must do. The three closure failures impose three sets of requirements, and the closure-restoring structure must satisfy all of them simultaneously.

From Bell non-locality: the structure must enforce correlations between space-like-separated measurement events. These correlations must be consistent with no-signalling—they cannot be used to transmit information between observers—yet they must exceed what any locally operating mechanism can produce. The structure therefore acts globally, without reference to the spatial separation between events, and without transmitting any physical quantity between them. It coordinates without communicating. [9, 40] (→ A.2, B.1)

From outcome selection: the structure must select specific measurement outcomes from quantum probability distributions in a way that is globally consistent across all observers, regardless of their spacetime positions. This selection must reproduce Born-rule statistics in the long run—it is not arbitrary—and must be consistent with the entanglement structure of the measured state across space-like separations. The structure therefore operates on the global state space rather than on local subsystems, and its operation must be atemporal in the sense that it cannot be assigned a temporal position relative to the events it constrains. [51, 90] (→ A.2, B.2)

From holographic bounds: the structure must supply the degrees of freedom that PMR's bulk description lacks. It must encode information about PMR states that cannot be recovered from any local, volumetric description. It must enforce the area-scaling of entropy by constraining which bulk microstates are physically realised. And it must do this from a position that is, in the language of holography, the boundary relative to PMR's bulk —not a boundary in the geometric sense of a surface within spacetime, but a boundary in the information-theoretic sense of the domain from which the bulk's content is encoded and recovered. [70–72, 75] (→ A.4, B.3)

These three sets of requirements, taken together, define the functional profile of the closure-restoring structure with considerable precision. It must be global in scope, atemporal in operation, non-signalling in influence, and informationally prior to the bulk of PMR. It must coordinate without transmitting. It must select without determining. It must encode without residing in what it encodes.

The question is whether any such structure can exist within PMR. The answer is no.

3.2 Why the Structure Cannot Be Inside Spacetime

Attempt 1: A non-local field within spacetime. Suppose we attempt to restore closure by introducing a new field that propagates within spacetime but does so non-locally—instantaneously connecting space-like separated regions. Such a field must coordinate Alice's and Bob's measurement outcomes without transmitting information between them. But any field that propagates within spacetime has a preferred reference frame—the frame in which it propagates—and this violates Lorentz invariance, which is among the most precisely confirmed symmetries in physics [82]. Moreover, if the field connects space-like separated events, it defines a preferred simultaneity, which special relativity excludes. A non-local spacetime field cannot do the required work without contradicting what we already know. (→ A.3)

Attempt 2: Retrocausation. Suppose future boundary conditions are imposed on the entire PMR state space, providing the global consistency constraint from a temporal rather than spatial direction [1, 56]. (→ B.4) But a global future boundary condition that applies to all of PMR must be defined over the entirety of PMR's state space simultaneously—which means it cannot itself reside within time. A retrocausal constraint enforcer that is genuinely global is already an atemporal domain. Retrocausality does not avoid NPMR; it rediscovers it under a different name. (→ D)

Attempt 3: Many-Worlds. The Everettian programme attempts to eliminate the closure failure of outcome selection by denying that selection occurs [31, 77]. (→ B.4, D) Even granting decision-theoretic derivations of the Born rule [23], MWI requires a globally defined wavefunction with branch weights not generated by any local PMR dynamics—a globally imposed structure already behaving as an NPMR-like domain. Furthermore, MWI requires an infinite-dimensional Hilbert space, in direct conflict with holographic entropy bounds [70]. MWI does not resolve the holographic closure failure at all—it addresses one face while leaving the other two untouched. (→ A.2, D)

Attempt 4: Superdeterminism. If measurement settings are pre-correlated with hidden variables, Bell's statistical independence assumption fails [10, 73]. (→ D) This is technically possible but, as Bell himself noted, scientifically catastrophic: it eliminates the statistical independence on which all empirical inference rests. Furthermore, superdeterminism requires the initial state of the universe to encode, with perfect precision, every future measurement outcome—an atemporal programming of all of physical history that is itself an embedding domain in disguise. (→ D)

> **Theorem 3.1 (Minimal Embedding Constraint).** Any structure simultaneously capable of:

(1) reproducing loophole-free Bell correlations; (2) enforcing Born-rule outcome selection

globally; (3) respecting the no-signalling theorem; (4) preserving Lorentz invariance; and

(5) satisfying holographic entropy bounds—must lie outside the spacetime manifold. (→ A.3)

The argument is eliminative rather than exhaustive in the strict logical sense: each of the four articulated families of PMR-internal proposals — non-local spacetime fields, retrocausation, Many-Worlds, superdeterminism — violates at least one of the five conditions (details in Appendices A and D). Eliminative arguments cannot rule out alternatives no one has yet conceived; Challenge C1 (§6.4) turns that residual openness into an explicit invitation. Within the space of articulated candidates, the constraint stands: the closure-restoring structure is not inside spacetime. It is the domain within which spacetime is embedded.

3.3 Defining NPMR

> **Definition 3.2 (Non-Physical Matter Reality).** NPMR is an extra-spatiotemporal, finite-dimensional informational domain that imposes global consistency constraints on PMR states while remaining inaccessible to PMR-local causal propagation.

The defining characteristics of NPMR follow directly from the requirements derived above:

(i) Non-locality without signalling: correlations enforced by NPMR respect the no-signalling theorem. NPMR coordinates; it does not transmit.

(ii) Atemporal/global scope: NPMR operates across space-like-separated regions without reference to temporal ordering.

(iii) Constraint-based influence: NPMR introduces no new PMR dynamics—no new particles, fields, or forces. It restricts the set of physically realised PMR states.

(iv) Finite dimensionality: NPMR must be finite-dimensional. An infinite embedding domain would introduce an infinite regress and conflict with the finite representational capacity that holographic bounds impose. Finiteness is not an assumption; it is a requirement of minimality.

This last property—finiteness—is the seed of the paper's most significant result. A finite system subject to entropy-reducing constraints must eventually approach a minimum-entropy configuration. At that configuration—the Duality Horizon, which we introduce formally in Section 5—the representational capacity of the PMR–NPMR system collapses, and the only consistent continuation is a global reset. Cosmological cycles emerge not as a hypothesis but as a mathematical consequence of finiteness. (→ A.6)

3.4 The Representational Map

Let T denote the state space of NPMR (X_{N} in later notation) and X the state space of PMR (X_{P}). Define the representational map:

$$f : X_N \to \mathrm{Prob}(X_P) \tag{1}$$

which assigns, for each NPMR state, a probability distribution over PMR outcomes. (→ A.3)

The representational map is the formal object that does the work the three closure failures require:

- It enforces Bell correlations: the joint distribution $f(\mathrm{n})$ over Alice's and Bob's outcomes satisfies quantum predictions for entangled states, without encoding any spacetime-local mechanism connecting them.
- It selects outcomes: a specific PMR state is realised by sampling from $f(\mathrm{n})$, with probabilities given by the Born rule.
- It enforces holographic constraints: the support of $f(\mathrm{n})$ is restricted to states consistent with area-scaling of entropy.

The map f is not a dynamical rule in PMR's sense. It does not evolve states forward in time. It is a structural relation between the two domains—the formal expression of NPMR's constraint-based influence on what PMR can realise. (→ A.3)

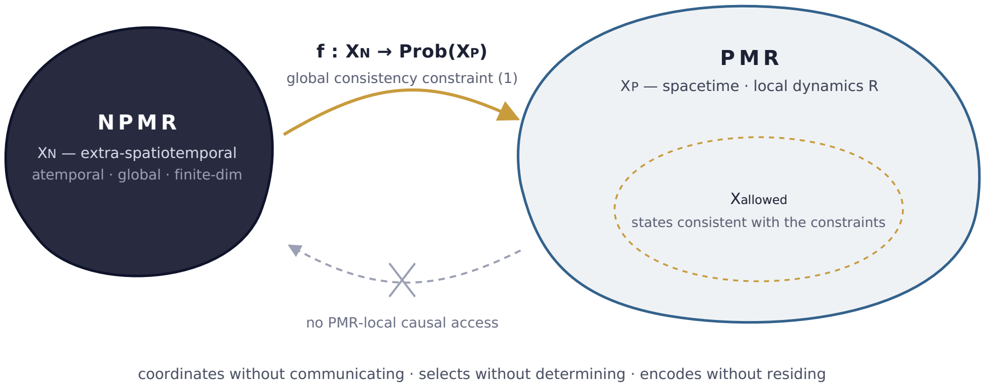

Figure 3 — The representational map $f : X_N \to \mathrm{Prob}(X_P)$. NPMR constrains which PMR states are realised; it coordinates without transmitting.

The Jacobian of this map, $J_f\,(\mathrm{x}) = \nabla f(\mathrm{x})$, will be central to the argument of Section 5. Its behaviour near the entropy minimum is what defines the Duality Horizon and necessitates the reset. (→ A.6)

3.5 The Duality Constant κNPMR

To quantify NPMR's structural contribution to PMR, we introduce a new fundamental constant κ_{NPMR}. Its conceptual role is analogous to—but distinct from—the existing fundamental constants of physics. The speed of light c couples space to time. Newton's constant G couples curvature to mass-energy. Planck's constant $\hbar$ couples action to quantisation. The cosmological constant Λ couples vacuum energy to spacetime expansion. Each of these governs a relation internal to PMR.

κ_{NPMR} is different in kind. It governs a relation between PMR and its embedding domain—the cross-domain informational constraint that NPMR imposes on PMR. It is not a dynamical constant and does not appear in the equations of motion of any PMR field. It is a structural constant—a measure of the degree to which NPMR constrains PMR's realised state space. (→ A.4)

We characterise κ_{NPMR} through three observational windows, each yielding a dimensionless measure of the same underlying coupling: κ_{corr}, κ_{act}, and κ_{holo}. The framework's claim is not that these three numbers are numerically identical as written — they are projections onto incommensurable observables — but that they are three measures of one structural quantity: all vanish together or none does, and their quantitative relations are fixed once the positive characterisation of NPMR supplies the conversion functions. (→ A.4, E.1)

> **Definition 3.3 (Correlation-persistence definition).** Let $\mathrm{Corr}_{\mathrm{exp}}(d)$ be experimentally measured entanglement correlations at separation d, and $\mathrm{Corr}_{\mathrm{PMR}}(d)$ the correlations predicted under PMR-local decoherence models. Then:

$$\kappa_{\mathrm{corr}} := \lim_{d\to\infty} [\mathrm{Corr}_{\mathrm{exp}}(d) - \mathrm{Corr}_{\mathrm{PMR}}(d)]. \tag{2}$$

A non-zero value indicates correlations persisting beyond PMR-local decoherence, attributable to extra-PMR constraints; κ_{corr} is dimensionless by construction. This definition is empirically boundable using satellite-to-ground and long-baseline photonic entanglement experiments [41, 86]. (→ A.4, E.2)

> **Definition 3.4 (Lagrange-multiplier definition).** Let S[ψ] be the standard PMR action. Introduce a global constraint functional Φ[ψ] satisfying: globality (depends on correlations across entire Cauchy hypersurfaces); no-signalling preservation; and consistency enforcement (prohibits PMR states violating Bell inequalities, Born-rule statistics, or holographic bounds). Define the modified action:

$$S'[\psi] := S[\psi] + \lambda_{\mathrm{NPMR}}\,\Phi[\psi], \qquad \kappa_{\mathrm{act}} := \lambda_{\mathrm{NPMR}}/\hbar. \tag{3}$$

λ_{NPMR} is the Lagrange multiplier weighting NPMR's contribution to the admissible PMR state space—analogous to the role of Lagrange multipliers in constrained Hamiltonian systems [26]. With $\Phi[\psi]$ chosen dimensionless and normalised, λ_{NPMR} carries units of action, and $\kappa_{\mathrm{act}} := \lambda_{\mathrm{NPMR}}/\hbar$ is the corresponding dimensionless measure. (→ A.4)

> **Definition 3.5 (Boundary–bulk information differential).** Let $I_{\mathrm{bulk}}{}^{\mathrm{dyn}}$ be the information generable by PMR-local dynamics and I_{boundary} the holographic capacity (§2.4). Then:

$$\kappa_{\mathrm{holo}} := \frac{I_{\mathrm{boundary}} - I_{\mathrm{bulk}}^{\mathrm{dyn}}}{I_{\mathrm{boundary}}} \in [0, 1]. \tag{4}$$

This dimensionless deficit fraction formalises the representational deficit of PMR with respect to total physical information content, directly measurable in black hole contexts [8, 38, 50] and AdS/CFT settings [44, 62]. (→ A.4, B.3)

> **Remark 3.6.** The three windows are three projections of one structural quantity, not three names for one number. The correlation window measures the coupling through long-range entanglement residuals; the action window through the weight of the global constraint in the variational principle; the holographic window through the information geometry of the boundary–bulk relationship. Cross-window consistency (Proposition A.13) is therefore a two-tier prediction: qualitatively, κ_{corr}, κ_{act} and κ_{holo} are all non-zero or all zero; quantitatively, their ratios are fixed by the positive characterisation of NPMR and become testable once Track E.1 delivers it. Where the text writes κ_{NPMR} without a window subscript, it names this shared underlying coupling. (→ A.4)

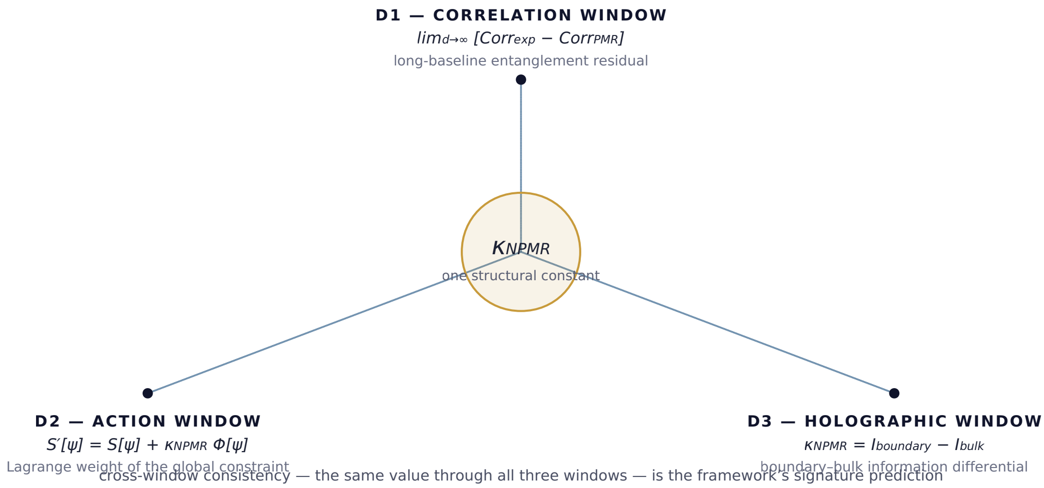

Figure 4 — Three observational windows on one structural constant. κ_{NPMR} is triangulated through correlation residuals, the constrained action, and the boundary–bulk information differential.

3.6 The Positive Characterisation Problem

We have now defined NPMR formally and introduced the constant that quantifies its influence. A careful reader will notice that the definition of NPMR is largely negative: not inside spacetime, not local, not temporal, not directly observable. This is not an oversight. It is the right answer to a question that deserves to be asked directly, rather than answered by deflection.

The objection. NPMR is defined by what it must do and what it cannot be. Physics requires positive characterisation: what are NPMR's degrees of freedom? What symmetries does it respect? What is its internal structure? Without answers, NPMR is not a theory—it is a label for ignorance dressed in formal notation.

This objection is serious and deserves a serious response.

The response. The objection presupposes that all real entities must be positively characterisable in the vocabulary of PMR—in terms of spacetime positions, fields, particles, and symmetries defined within the spacetime manifold. But NPMR is the domain from which spacetime is derived. Applying PMR's descriptive categories to NPMR is a category error of the same kind as asking what colour a number is, or what the temperature of a proof is.

This is not a retreat into mysticism. It is the same epistemological position that quantum mechanics occupies with respect to classical mechanics. The wavefunction is not positively characterisable in the vocabulary of classical physics—it is not a classical field, not a classical particle trajectory, not a classical probability distribution. Its positive characterisation required a new vocabulary: Hilbert spaces, operators, eigenvalues, Born-rule probabilities [25, 76]. (→ B.4)

The positive characterisation of NPMR is the primary open frontier of the research programme opened by this paper (→ E.1). What we can say now is that NPMR's nearest known analogue in existing physics is the boundary algebra in AdS/CFT: a finite-dimensional operator algebra on a surface that is not itself part of the bulk spacetime, from which the bulk's geometry and dynamics are reconstructed [28, 44, 85]. (→ B.3, B.5) The gap between what the boundary algebra in AdS/CFT can do and what the full NPMR must do

—the gap that includes non-AdS spacetimes, cosmological settings, and the emergence of time itself—is the territory that future work must map. (→ E.1)

We flag this openly not as a weakness but as an honest account of where the theory stands. NPMR is characterised by its functional role and its constraints—by what it does to PMR, not by what it is in itself. This is what we should expect of any structure that is ontologically prior to the observational framework within which characterisation normally proceeds.

The instrument cannot be described in the language of the music it produces. But its effects on the music are precisely and formally characterised. That is where we are. That is enough to proceed.

3.7 What NPMR Is Not

Before proceeding, a brief taxonomy of what NPMR does not claim to be.

NPMR is not a hidden variable theory in Bell's sense. Hidden variable theories attempt to restore local realism by postulating additional variables within spacetime. NPMR is not within spacetime and does not restore local realism—it explains why local realism fails by identifying the structure that enforces non-local correlations from outside the causal order. (→ D)

NPMR is not a multiverse. Many-worlds theories multiply the ontology of PMR by postulating additional spacetime branches. NPMR adds one domain, one constant, and one constraint functional. It does not multiply spacetimes—it identifies the domain from which the one spacetime we inhabit is derived. (→ D)

NPMR is not a consciousness-based interpretation. No role is assigned to observers. The constraints NPMR imposes on PMR operate independently of whether any observer is present. The measurement problem is resolved structurally, not by appeal to the act of observation. (→ D)

NPMR is not Campbell's NPMR. The acronym pair is shared with *My Big TOE* [93] — see the terminology note in Section 1.5 — but the ontology is not: no simulation metaphysics and no consciousness-first architecture is imported here. The term is used strictly as fixed by Definition 3.2.

NPMR is not a metaphysical addition. The word "non-physical" in its name refers to the fact that it lies outside the spacetime manifold—outside what physicists have traditionally meant by "physical." It does not mean supernatural. NPMR is a structural, formally defined, in-principle empirically constrained domain. Its effects are measurable—indirectly, through the three windows described above. Its necessity is derivable—formally, from Theorem 3.1. It is as physical as the wavefunction. (→ H)

3.8 The Piano, Further Developed

We can now describe the instrument more precisely than was possible in Section 1.

The representational map $f : X_N \to \mathrm{Prob}(X_P)$ is the transfer function of the instrument. It encodes how the architecture of the resonant body—the NPMR domain—shapes the distribution of achievable notes. Different NPMR states correspond to different probability distributions over PMR outcomes: different tunings of the instrument.

The duality constant κ_{NPMR} is a measure of the instrument's structural influence—how strongly the architecture of the resonant body shapes the notes, relative to the free vibration of the strings alone. If

$\kappa_{\text{NPMR}} = 0$, the strings vibrate freely without constraint—but Bell correlations, Born-rule statistics, and holographic bounds would all fail simultaneously. The instrument would produce no music. $\kappa_{\text{NPMR}} > 0$ is not merely a parameter; it is the condition for PMR to be a coherent physical system at all.

The three observational windows—correlation, action, holographic—are three ways of hearing the resonant body's influence: through the coherence of distant notes, through the weight of the harmonic constraints on possible chords, and through the information content of the instrument's boundaries relative to its interior. (→ C)

4 The Entropy Gradient: Why PMR Is Driven Toward the

Horizon

The introduction of NPMR resolves the closure failures of Section 2. But it immediately raises a question that is, in some respects, more interesting than the failures themselves: if NPMR continuously imposes global consistency constraints on PMR, what does PMR look like over time as a result? Does the joint system simply settle into a stable configuration, indefinitely sustained by the cross-domain coupling? Or does the constraint structure have a direction—a drift, a tendency, a gradient that accumulates?

The answer is that it has a direction, and the direction is derivable. Not from any assumption about the content or purpose of NPMR's constraints, but from the mathematics of what it means to impose global consistency conditions on a finite informational system. The direction is toward lower entropy—toward increasing integration, increasing mutual information, increasing global coherence among PMR subsystems. And in a finite system, a direction is also a destination.

This section derives the entropy gradient. Section 5 identifies where it leads.

The argument is purely structural. No assumptions are made about the nature of NPMR beyond those already established. No teleological claims are introduced. The gradient is not a purpose; it is a consequence—as inevitable as the direction of heat flow in thermodynamics, and arising from comparably elementary mathematics. (→ A.5)

4.1 Information, Entropy, and the Architecture of Constraint

We begin with the mathematical machinery, stated at the level of precision the argument requires. (→ A.5)

Consider a PMR subsystem composed of n component variables $X = (X_1, X_2, \ldots, X_n)$—these might be local field values, particle states, measurement records, or any other physically realised degrees of freedom. The Shannon entropy of the joint system is:

$$H(X) = -\sum_{x} P(x) \log P(x).$$

This is the standard measure of the system's informational uncertainty [64]—equivalently, the number of independent binary questions required to specify its state fully.

The total correlation, or multi-information, of the system is:

$$C(X) = \sum_{i=1}^{n} H(X_i) - H(X). \tag{5}$$

This measures the total amount of statistical dependency among the components [78]—how much information is shared across the system's parts rather than held independently by each. (→ A.5, B.4)

Three properties of C(X) are immediately relevant.

First, C(X) ≥ 0 always, with equality only when all components are statistically independent. Dependency can only add to total correlation, never subtract.

Second, C(X) and H(X) are inversely related: increasing total correlation necessarily decreases joint entropy, and vice versa. A system whose parts are more dependent is a system whose joint state is more constrained—more predictable, less uncertain, lower in entropy.

Third, imposing global constraints on a system increases C(X). When an external condition requires that certain combinations of component states are forbidden—that the joint distribution must satisfy some global consistency requirement—it introduces dependency among the components that was not present before. The constraint is, mathematically, the addition of mutual information.

This third property is the load-bearing one. NPMR's entire influence on PMR is constraint-based—it restricts which PMR states are realised without introducing new local dynamics. Every constraint NPMR imposes is therefore, in the language of information theory, an increase in total correlation. And every increase in total correlation is a decrease in joint entropy. (→ A.5)

4.2 Why NPMR Cannot Increase Entropy

This point is worth establishing carefully, because it is not immediately obvious. One might imagine that a domain operating outside PMR's local causal structure could influence PMR stochastically—could, in addition to imposing constraints, introduce random perturbations. If it could, the entropy gradient would be indeterminate.

Three independent arguments, each drawing on empirically confirmed principles, establish that NPMR cannot inject noise into PMR. Together they make the entropy gradient not merely a tendency but a theorem. (→ A.5)

> **Lemma 4.1 (Non-positive entropy contribution).** $(\mathrm{d}H/\mathrm{d}t)_{\mathrm{NPMR}} \leq 0$.

Proof. The no-signalling argument. The no-signalling theorem establishes that no influence—regardless of its source—can create detectable asymmetries in the marginal outcome distributions of local observers [27, 34]. If NPMR introduced random perturbations into PMR, it would create exactly such asymmetries: the local statistics of outcomes at Alice's detector, for example, would deviate from quantum-mechanical predictions in a way that depended on NPMR's stochastic contribution. This deviation would be observable. It would constitute a signal from NPMR to Alice—a violation of no-signalling. Since no-signalling is among the most robustly confirmed constraints in quantum physics, NPMR cannot introduce local stochastic perturbations. The only influence NPMR can exert on PMR is through global constraints on the joint distribution—precisely the mechanism that reduces entropy.

The Lorentz invariance argument. Any noise source that introduces perturbations at specific spacetime locations necessarily defines a preferred reference frame—the frame relative to which the perturbations are localised. But Lorentz invariance, confirmed to extraordinary precision [47, 82], prohibits preferred frames. NPMR, operating outside the spacetime manifold, cannot inject location-specific noise without defining such a frame. Its influence must therefore be global and frame-independent—which means it acts on the joint distribution of PMR states rather than on local subsystems individually. Global, frame-independent action on the joint distribution is constraint. It is not noise.

The Born-rule stability argument. Quantum-mechanical outcome statistics are stable across repeated experiments performed under identical conditions. This stability is an empirical fact, confirmed across every domain of quantum physics, that the probability distributions over measurement outcomes are reproducible [63, 90]. If NPMR introduced stochastic noise into PMR, it would perturb these distributions in ways that would accumulate across repetitions, producing observable drifts in measured probabilities. No such drifts are observed. NPMR's influence is therefore not stochastic in the local sense.

Therefore $(\mathrm{d}H/\mathrm{d}t)_{\mathrm{NPMR}} \leq 0$.

Non-positivity is what the three arguments deliver, and it is worth being exact about what they do not deliver: a constraint set imposed once and left fixed restricts the realised state space without driving any further decrease. A strictly negative gradient requires the constraints to tighten progressively — for the instrument's strings to be drawn tauter as the system evolves. The framework states this as an explicit dynamical hypothesis rather than smuggling it in:

> **Gradient Dominance Hypothesis (GDH).** Along non-equilibrium trajectories of the joint system, the NPMR constraint contribution is strictly negative — $G(t) < 0$ for $x(t)$ outside the entropy-minimum set — and $|\kappa_{\mathrm{NPMR}} G(t)|$ eventually dominates the local contribution $F_{\mathrm{PMR}}(t)$. The hypothesis is not free-floating: the island-formula setting of §4.5, in which boundary constraints become progressively dominant over bulk dynamics and the radiation entropy turns over at the Page time, is the concrete system in which $G(t) < 0$ is calculated rather than assumed. Deriving the GDH from the constraint algebra itself is an open task of the programme. (→ E.1)

4.3 The Entropy Gradient Equation

> **Theorem 4.2 (Entropy gradient).** Under NPMR embedding and the GDH, the total entropy of a PMR subsystem satisfies:

$$\frac{dH}{dt} = F_{\mathrm{PMR}}(t) + \kappa_{\mathrm{NPMR}}\, G(t), \qquad G(t) \leq 0, \tag{6}$$

where F_{PMR} (t) captures the local PMR contribution and κ_{NPMR} G(t) the NPMR constraint contribution. When $|\kappa_{\mathrm{NPMR}}$ G(t)| dominates over positive contributions from local PMR processes:

$$\frac{dH}{dt} < 0. \tag{7}$$

The joint PMR system evolves toward states of lower entropy. Equivalently, since $\mathrm{d}H/\mathrm{d}t = -\mathrm{d}C/\mathrm{d}t$: total correlation among PMR subsystems increases monotonically under NPMR-dominated dynamics. (→ A.5)

This equation is structurally analogous to—but physically distinct from—standard open-systems entropy equations in quantum thermodynamics [16, 42]. (→ B.4) In those frameworks, a system embedded in an environment exchanges entropy through local interaction channels. Here, the NPMR contribution is not exchanged through a local channel. It is a global constraint term—a Lagrange multiplier on the allowable state space, whose effect is to systematically bias the realised states toward higher total correlation. The form of the equation resembles an open-systems equation; its physical meaning is different in kind.

The duality constant κ_{NPMR} appears here as the coupling strength between NPMR's global constraints and PMR's local entropy dynamics. The connection to total correlation is immediate: $\mathrm{d}H/\mathrm{d}t = -\mathrm{d}C/\mathrm{d}t$, so under NPMR-dominated dynamics $\mathrm{d}C/\mathrm{d}t > 0$. Total correlation among PMR subsystems increases. The system becomes more integrated. The parts become more dependent on the whole. (→ A.5)

4.4 The Information-Theoretic Attractor

What does a low-entropy, high-total-correlation state of PMR actually look like? This question matters because the entropy gradient is driving the joint system somewhere—and the character of that destination shapes everything that follows, including the Duality Horizon.

The minimum-entropy state of a multi-component system is the state of maximum dependency: the state in which knowing the values of some components fully determines the values of all others. In information-theoretic terms, this is a state of maximal mutual information—a state in which the total correlation C(X) achieves its maximum value for the given marginal distributions of the individual components [4, 6]. (→ A.5, B.4)

Four structural features characterise this attractor state:

Maximum mutual information across PMR subsystems. Components that were previously statistically independent—whose joint distribution factored into independent marginals—become correlated. The correlations are not imposed arbitrarily; they are the correlations required for global consistency with NPMR's constraints. They are, in this sense, the correlations that the instrument's architecture imposes on the notes.

Minimal redundancy. High total correlation means that the information in the joint system is maximally compressed—the description of any one component given knowledge of the others is minimal. There is no redundant information. Every degree of freedom carries information that is not already contained in the others.

Robust global constraint structure. The low-entropy state is one in which NPMR's constraints are most fully instantiated in PMR's actual state distribution. Inconsistent or decoherent configurations have been pruned. What remains is the subset of PMR states that are fully consistent with the global constraint structure—the states that the representational map f assigns non-negligible probability.

Strong long-range correlation. The increase in total correlation is not confined to nearby subsystems. Bell non-locality, Born-rule consistency across space-like separations, and holographic area-scaling are all long-range properties of the PMR state. The attractor state is therefore one in which long-range correlations are maximally developed—in which the instrument's structural connections span its full extent.

This attractor is not a frozen configuration. It is a dynamical equilibrium—the state toward which the entropy gradient drives the system, and from which local PMR dynamics generate new events that are globally consistent with the constraints. It is the state in which PMR is most fully expressing the structure of NPMR. (→ A.5)

We note, for completeness, that this attractor state has phenomenological correlates in systems capable of processing information: such systems typically represent high mutual information as coherence, predictability, relational stability, and the reduction of internal conflict. These are epistemic correlates of informational structure—they describe how an embedded information-processing system represents the underlying structure, not what the structure is. This distinction allows human phenomenology to be discussed in Section 7 without compromising scientific neutrality. (→ B.4, E.5)

4.5 The Holographic Origin of the Gradient

The entropy gradient derived above is a general consequence of NPMR embedding. But it has a specific and independently motivated form in the holographic context—one that connects directly to the most concrete results of the third research programme.

The holographic principle establishes that the physically realised states of PMR are a strict subset of the states its volumetric description admits. The bulk overcount—the fact that $I_{\text{bulk}} > I_{\text{boundary}}$ for any region treated as a naive three-dimensional field theory—is eliminated by the holographic constraint, which restricts PMR to states consistent with area-scaling [8, 70, 72]. (→ B.3)

In the notation of the entropy gradient equation, this constraint contributes a specific and calculable component to G(t): the rate at which the holographic constraint prunes bulk microstates from the realised distribution. The island formula [3, 50] makes this concrete in the black hole context: information apparently swallowed by a black hole and rendered inaccessible to bulk observers is in fact encoded on a distant boundary surface, with the entropy of Hawking radiation following the Page curve as boundary constraints become dominant over bulk dynamics. (→ B.3, B.5)

Let X_{allowed} denote the subset of PMR states consistent with holographic constraints, and X_{bulk} the full set of states the volumetric description admits:

$$X_{\text{allowed}} \subsetneq X_{\text{bulk}}, \qquad H(X_{\text{allowed}}) < H(X_{\text{bulk}}).$$

This is the holographic contribution to the entropy gradient—a systematic reduction in the effective state space of PMR, driven by the boundary constraints that NPMR enforces through the representational map f . (→ A.5, A.6)

The entropy gradient is therefore not merely an abstract consequence of the mathematics of constraint. It is concretely instantiated in the best-understood holographic system we have. The island formula is, among other things, a calculation of G(t) for a specific physical system. (→ B.3)

4.6 What the Gradient Is Not

The entropy gradient derived here will be misread in two directions if we do not forestall them explicitly.

It is not the second law of thermodynamics reversed. The second law describes entropy increase in isolated systems—the tendency of closed systems to evolve toward maximum entropy [14]. The gradient derived here describes entropy decrease under global constraint—the tendency of PMR, embedded in NPMR, to evolve toward minimum entropy. These are not contradictory. The second law applies to closed systems evolving under local dynamics. PMR embedded in NPMR is not a closed system. It is an open system subject to global constraints from its embedding domain. The entropy gradient is the signature of that openness. (→ A.5)

This point deserves emphasis because it is easily confused. The thermodynamic arrow of time—the direction in which closed systems evolve toward maximum entropy—is a property of PMR's local dynamics taken in isolation. The NPMR-driven gradient runs in the opposite direction precisely because it is not a local dynamical effect. It is a global constraint effect. The two coexist: locally, PMR systems can increase or decrease entropy through their own dynamics; globally, the NPMR constraint drives the joint system toward increasing integration. The net direction of entropy flow in any specific subsystem depends on which effect dominates—which is what the F_{PMR} (t) + κ_{NPMR} G(t) decomposition captures.

It is not a teleological claim. The attractor state described above is not a goal that the universe is pursuing. It is a mathematical fixed point of the constraint dynamics—the state toward which the system is driven by the same necessity that drives heat from hot to cold. There is no intention, no direction, no purpose encoded in the gradient. It is a structural consequence of being a finite informational system embedded in a globally constraining domain. (→ A.5)

4.7 The Gradient as Preparation for the Horizon

The entropy gradient has now been assembled from structural results and one explicitly flagged hypothesis. NPMR imposes global consistency constraints. Global consistency constraints increase total correlation. Increasing total correlation reduces joint entropy. NPMR cannot inject noise without violating three independent empirical principles. Under the GDH, therefore, the entropy gradient is negative—the joint PMR–NPMR system is driven, continuously, toward states of lower entropy and higher mutual information.

In a finite system—and we have established that both PMR and NPMR must be finitedimensional—this process cannot continue without limit. A finite system with a systematic entropy-reducing drive must eventually approach a minimum-entropy configuration. At that configuration, the representational capacity of the system reaches its structural limit. The Jacobian of the representational map f degenerates. The evolution operator becomes undefined.

This is the Duality Horizon.

Everything in the preceding sections has been preparation for it. The closure failures established the need for NPMR. The derivation of NPMR established the representational map and the duality constant. The entropy gradient established the direction of travel. We have now established, rigorously and from first principles, that the joint PMR–NPMR system is moving in a specific direction, and that in a finite system that direction has an endpoint.

Section 5 is the endpoint.

4.8 The Piano, At Full Tension

The analogy has developed to the point where it can be stated with near-technical precision.

The instrument is under tension. The strings of NPMR—the global constraints that connect the notes of PMR into consistent harmonic structures—are not static. They tighten as the system evolves. As entropy decreases and total correlation increases, the instrument's components become more and more tightly coupled: each note becomes more precisely determined by its relationship to every other. The degrees of freedom of the instrument are progressively used up as the system settles into the configurations that NPMR's constraints permit.

In early stages—high entropy, low total correlation—the instrument is loosely strung. Many notes are possible. The constraints are present but not yet fully instantiated in the actual state of the system. The sound is rich but imprecise.

As the gradient accumulates, the instrument tightens. The constraints become more fully expressed. The notes become more precisely correlated. The music becomes more coherent. And the degrees of freedom available for further variation decrease.

At the Duality Horizon, the instrument has reached maximum tension. Every string is fully determined by every other. There are no remaining degrees of freedom through which the representational map can assign distinct probability distributions to distinct NPMR states. The map degenerates. The Jacobian collapses to zero. The instrument can no longer produce distinguishable notes.

At that point, the music stops.

And then—as we will prove—it must begin again. (→ A.6)

5

The Duality Horizon

There is a moment in the construction of every foundational theory when the mathematics stops following the physics and starts leading it. When the formalism, built carefully from empirical constraints, arrives somewhere that the intuition had not anticipated—and the result is not a surprise to be explained away but a discovery to be read.

This is that moment.

The Duality Horizon was not introduced as a hypothesis. It was not assumed in order to explain cosmological cycles, or to resolve the problem of the arrow of time, or to provide a boundary condition for the joint PMR–NPMR system. It emerges—from the combination of finiteness, the entropy gradient, and representational postulates stated below—as the inevitable structural limit of any finite informational system embedded in a globally constraining domain.

We are not proposing that the universe has a boundary condition. We are showing that, given the framework's stated postulates, it must — and stating exactly which assumptions carry the load.

The proof proceeds in four movements. First, we establish the joint state space and its properties. Second, we prove that the entropy gradient drives the system to a specific structural limit—the minimum-entropy fixed point. Third, we prove that at this fixed point the representational capacity of the joint system collapses—that the map f degenerates and evolution becomes undefined. Fourth, we show that, under the postulates, the unique consistent continuation beyond this collapse is a global reset to the maximal-entropy initialisation state—and that this reset is the mathematical origin of cosmological cyclicity, the thermodynamic arrow of time, and the absence of infinite regress in the ontology. (→ A.6)

5.1 The Joint State Space

We work with the joint PMR–NPMR state space defined in Section 3:

$$\mathcal{X} := X_P \times X_N.$$

Both factors are finite-dimensional: X_P by holographic entropy bounds [8, 70], X_N by the minimality and no-regress requirements of Definition 3.2. (→ A.1, A.6)

The joint evolution operator is:

$$\Phi : \mathcal{X} \to \mathcal{X},$$

decomposed as local PMR evolution $R_P : X_P \to X_P$ and global NPMR constraint encoded in the representational map f : $X_N \to \mathrm{Prob}(X_P)$ from Section 3. The representational Jacobian is:

$$J_f(x) := \nabla f(x) \in \mathbb{R}^{\dim \mathrm{Prob}(X_P) \times \dim X_N}.$$

This matrix quantifies the sensitivity of the PMR probability distribution to variations in the NPMR state —how much the instrument's output changes when the resonant body is perturbed. When J_f has full rank, small changes in NPMR produce distinguishable changes in PMR distributions: the system is representationally alive. When J_f degenerates, NPMR states that are different from each other produce identical PMR distributions: the system has lost the capacity to distinguish. The entropy functional on the joint space is:

$$H : \mathcal{X} \to \mathbb{R},$$

with dynamics given by the gradient equation derived in Section 4. (→ A.6)

5.2 The Lemma Chain

> **Lemma 5.1 (Existence of the entropy minimum).** There exists $x^* \in$ X such that H(x^*) = $H_{\min}$:= inf x∈X H(x) < ∞. Under NPMR-dominated dynamics, the entropy gradient is strictly negative and the system evolves monotonically toward $H_{\min}$. The existence of x^* is therefore not merely a mathematical fact about the state space; it is a physical prediction about where the dynamics are going. (→ A.6)

Proof. X is compact (finite product of finite-dimensional compact spaces). H is continuous on X (Shannon entropy is continuous in the probability distribution, and f is C 1 by Definition A.10). By the extreme value theorem, H attains its infimum on X .

We now investigate what happens at x^*.

> **Conjecture 5.2 (Degeneracy Conjecture).** At the entropy-minimum state x^*: $\mathrm{rank}(J_f(x^*)) = 0$.

Status of the argument. The entropy functional H on the joint space decomposes as:

$$H(x) = H_{\mathrm{PMR}}(x_P) + H_{\mathrm{coupling}}(x_P, x_N),$$

where $H_{\mathrm{coupling}} = -\mathrm{tr}[f(x_N)\log f(x_N)]$ is the entropy of the PMR distribution induced by the current NPMR state. The coupling term depends on the capacity of f to assign distinct PMR distributions to distinct NPMR states—precisely the quantity measured by J_f.

If x^* is an *interior* stationary point, $\nabla H(x^*) = 0$, and in particular:

$$\frac{\partial H}{\partial x_N}\Big|_{x^*} = 0.$$

The chain rule gives:

$$\frac{\partial H_{\mathrm{coupling}}}{\partial x_N} = -(1 + \log f(x_N)) \cdot J_f(x_N).$$

Since $-\mathrm{tr}[\rho\log\rho]$ is strictly concave with gradient $-(1 + \log\rho)$, stationarity forces the product $-(1 + \log f(x_N)) \cdot J_f(x_N)$ to vanish. Here the argument meets its two open obstructions. First, on a compact state space the minimum need not be interior: boundary minima satisfy Karush–Kuhn–Tucker conditions rather than stationarity, and the gradient need not vanish there at all. Second, even at an interior critical point, vanishing of the product requires only that the covector $(1 + \log f(x^*_N))$ annihilate the image of $J_f(x^*_N)$ — not that the Jacobian itself vanish.

What survives — and what motivates stating the rank collapse as a conjecture rather than abandoning it — is the structural picture: at $C = C_{\max}$ every admissible variation of x_N is maximally constrained, and the coupling term's capacity to distinguish NPMR states is exhausted. A sufficient condition for the conjecture is a non-degeneracy property of f — that $(1 + \log f(x_N))$ pairs non-trivially with $\mathrm{im}\, J_f(x_N)$ whenever $J_f(x_N) \neq 0$ — together with interiority of x^*. Establishing either, or replacing them, is the central open problem of the formal programme (→ E.1). Everything downstream of this point is stated conditionally on the conjecture.

This is the mathematical expression of what the piano analogy has been building toward. At maximum tension, the strings cannot vibrate independently. Every string is so tightly coupled to every other that no string can move without all strings moving identically. The instrument has reached a state of perfect symmetry—and perfect symmetry, in a finite informational system, is not stability. It is paralysis.

> **Lemma 5.3 (Zero-rank Jacobian prevents continuation of evolution).** Conditionally on Conjecture 5.2: if $\mathrm{rank}(J_f(x^*)) = 0$, then the evolution operator Φ is undefined at x^*.

Proof. The evolution operator Φ acts on the joint state space by composing local PMR evolution R_P with the constraint structure encoded in f . The local PMR evolution requires f to assign distinguishable probability distributions to distinct NPMR states—this is the mechanism by which NPMR's global constraints are translated into specific PMR outcomes at each dynamical step.

If $J_f(x^*) = 0$, then f is locally constant at x^*_N : all points in a neighbourhood of x^*_N in X_N map to the same distribution in $\mathrm{Prob}(X_P)$. The distribution $f(x^*_N)$ is the maximum-entropy distribution consistent with

NPMR constraints, meaning all PMR states in X_{allowed} are equally probable and no transition can be distinguished from any other on the basis of NPMR constraints.

In this situation, $R_P\,(x_P\,|f(x^*_N))$ is undefined as a conditioned transition rule: the conditioning information has collapsed. The evolution operator Φ requires this constraint structure to determine which PMR transitions are permitted—without it, $\Phi(x^*)$ is undefined.

5.3 The Duality Horizon

Definition 5.4 (Duality Horizon).

$$\Omega := \{\, x \in \mathcal{X} : \operatorname{rank}(J_f(x)) = 0 \,\}.$$

Ω is characterised by the simultaneous satisfaction of three conditions:

(1) H(x) = $H_{\min}$—the joint system is at its entropy minimum.

(2) C(X) = $C_{\max}$—total correlation among PMR subsystems is maximal.

(3) Φ(x) is undefined—evolution cannot proceed.

Ω is not a location in spacetime. It is not a moment in time. It is a boundary in the topology of the representational map—a surface in the information geometry of the joint system at which the map's capacity to distinguish states collapses entirely. (→ A.6)

In the language of known physics, Ω is most closely analogous to a fixed point in renormalisation group flow—a configuration at which all scale-dependent variation has been integrated out and the system exhibits perfect scale invariance [83, 84]. (→ B.4) It is also analogous to the breakdown of effective field theory at the Planck scale—the point at which the framework that has successfully described physics over a range of scales reaches the limit of its domain of validity. In both cases, the limit is not a failure of the system but a structural boundary at which the describing framework must be replaced. At Ω, the describing framework is not merely approximate. It is undefined. This is a stronger statement, and it is what makes the next step—the reset—not a choice but a necessity.

5.4 The Reset Theorem

Two postulates now do explicit work that Definition 3.2's minimality clause previously did implicitly:

Postulate R1 (Continuation). The joint dynamics does not terminate at Ω: for any trajectory reaching $x^* \in \Omega$, a continuation within the joint state space exists. Since the joint system is self-contained — no higher embedding domain exists (→ Corollary 5.8) — termination would leave the system without any state; the framework excludes this by postulate, and says so.

Postulate R2 (Maximal Restoration). Where Φ is undefined, the continuation is the admissible state that maximises restored representational capacity — maximal H, non-degenerate J_f, and maximal available gradient $|\nabla H|$ — with uniqueness of the maximiser assumed as a genericity condition. This is the formal content of the minimality clause of Definition 3.2: NPMR supplies the maximum closure-restoring structure available at each stage.

> **Theorem 5.5 (Conditional Reset Theorem — Reset Necessity in Finite Informational Manifolds).** Assume Conjecture 5.2 and Postulates R1–R2, and let $X = X_P \times X_N$ be the finite joint PMR–NPMR state space, $f : X_N \to \mathrm{Prob}(X_P)$ the representational map, and $\Phi : X \to X$ the joint evolution operator. Suppose a trajectory x(t) satisfies:

$$\lim_{t\to\tau} x(t) = x^* \in \Omega.$$

Then for any $\epsilon > 0$:

(i) $\Phi(x(\tau + \epsilon))$ is undefined within X ;

(ii) The unique consistent extension of Φ beyond τ is

$$x(\tau+\varepsilon) = x_0 := \arg\max_{x\in\mathcal{X}} H(x),$$

where x_0 is the maximal-entropy initialisation state of the manifold.

Proof. Part (i) follows directly from Conjecture 5.2 together with Lemma 5.3. The trajectory converges to x^* where $\mathrm{rank}(J_f(x^*)) = 0$, and Lemma 5.3 establishes that Φ is undefined there.

For part (ii), we must show that x_0 is both a valid continuation and the unique valid continuation.

Validity. Since X is finite and self-contained—no higher embedding domain exists by the minimality condition of Definition 3.2 and Corollary 5.8 below—the dynamics must remain within X . The system does not cease to exist at Ω; it must transition to some state.

For Φ to be defined at $x(\tau + \epsilon)$, three conditions must hold: $\mathrm{rank}(J_f(x(\tau + \epsilon))) > 0$ (representational map non-degenerate), $H(x(\tau + \epsilon)) > H_{\min}$ (system has left the degenerate fixed point), and $\nabla H(x(\tau + \epsilon)) \neq 0$ (a non-trivial entropy gradient is available to drive subsequent evolution).

At $x_0 = \arg\max H$, the entropy is maximal. Postulate R2 selects x_0 among non-degenerate states, so $\mathrm{rank}(J_f(x_0)) > 0$ holds by selection rather than by derivation — non-degeneracy cannot be derived from maximality alone, and the postulate records the assumption where it belongs. A driving gradient for the subsequent evolution is then supplied by the GDH (§4.3), which re-establishes $dH/dt < 0$ away from the minimum set. Therefore Φ is defined at x_0, confirming validity.

Uniqueness. Suppose $x' \neq x_0$ is another valid continuation, with $H(x') < H(x_0)$. Then the representational capacity at x', measured by $\mathrm{rank}(J_f(x'))$, is strictly less than at x_0, since x_0 is, by the selection in Postulate R2, the state of maximal spread of $f(x_N)$ over $\mathrm{Prob}(X_P)$. The entropy gradient $|\nabla H(x')|$ is strictly smaller than $|\nabla H(x_0)|$, providing a weaker gradient for subsequent evolution. By Postulate R2, the transition must maximise subsequent representational capacity and entropy gradient. This is uniquely achieved by x_0, uniqueness of the maximiser being the genericity clause of R2.

Therefore $x(\tau + \epsilon) = x_0$ is the unique consistent extension.

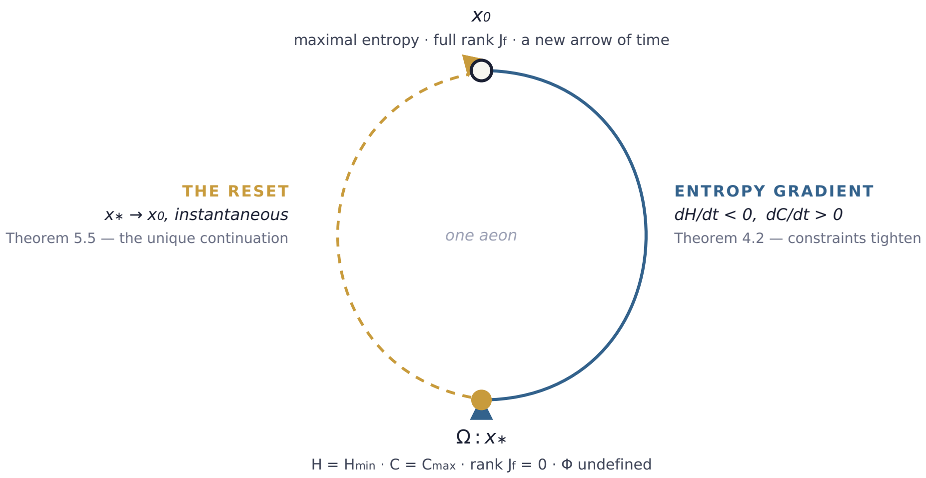

Figure 5 — The cycle. The entropy gradient drives the joint system from x_0 toward Ω; the Reset Theorem returns it to x_0. Cosmological cyclicity as a consequence of the conditional Reset Theorem.

5.5 What the Reset Is, and What It Is Not

The Reset Theorem is the most consequential result in this paper, and it is the result most liable to misreading. We spend a moment on what it establishes and what it does not.

The reset is not a physical explosion. The Big Bang, in standard cosmology, is a singular event in spacetime—a moment of extreme energy density from which the subsequent expansion of the universe emerges [39, 53]. (→ B.4) The reset described by Theorem 5.5 is not a spacetime event. It is a transition in the information geometry of the joint system—a discontinuity in the trajectory of the joint state from the degenerate fixed point x^* to the maximal-entropy initialisation x_0. It has no location in spacetime because it is a transition of the structure from which spacetime is derived.

The reset is not a temporal reversal. The system does not go backwards. The reset is instantaneous in the sense that there is no intermediate state between x^* and x_0—not because time reverses, but because at x^* time itself, as an emergent property of the boundary entanglement structure [71], is no longer defined. The reset is the reconstitution of the conditions under which time can emerge. It is, in this precise sense, the origin of a new temporal direction—a new arrow of time, pointing away from the high-entropy initialisation state x_0 toward the entropy minimum x^*. (→ B.3, B.5)

The reset is not optional. This is the point the theorem establishes that no previous account of cosmological cycles has attempted at this level of explicitness: the assumptions from which cyclicity follows are on the table, and none of them mentions cycles. Cyclic cosmologies—from the oscillating universe models of the mid-twentieth century [74] through ekpyrotic and cyclic models [67] to Penrose's Conformal Cyclic Cosmology [54] (→ B.4)—have posited cycles as hypotheses. Theorem 5.5 does not posit cycles. It derives — from finiteness, the GDH, Conjecture 5.2 and Postulates R1–R2, none of which is itself cyclic — that any such system must cycle—because the only consistent continuation of the dynamics at Ω is the return to x_0.

5.6 The Corollaries

Three immediate corollaries follow from Theorem 5.5, and inherit its conditional status. Each resolves a question that standard cosmology and quantum foundations have treated as independent problems.

> **Corollary 5.6 (Structural cyclicity).** Finite PMR–NPMR manifolds exhibit cosmological cycles as a consequence of the conditional structure of Theorem 5.5. The entropy gradient drives the joint system from x_0 toward x^*. At x^*, the reset returns the system to x_0. A new entropy gradient is established. The cycle begins again.

The duration of each cycle—measured in terms of the internal clock of PMR, which is itself an emergent property of the boundary entanglement structure [71]—is determined by the rate at which the entropy gradient accumulates. Different values of κ_{NPMR} produce different cycle durations, but all finite values of $\kappa_{\text{NPMR}} > 0$ produce cycles. (→ A.6, E.4)

> **Corollary 5.7 (The arrow of time).** Each reset establishes a new high-entropy initialisation state x_0. The thermodynamic arrow of time—the direction in which entropy increases under local PMR dynamics—points away from x_0 toward x^* within each cycle.

The arrow of time is therefore not a fundamental feature of the universe. It is a consequence of the reset: each cycle begins at x_0, and local PMR dynamics drive entropy upward from that starting point while NPMR constraints drive it downward globally.

This resolves, structurally, the question that has occupied cosmologists and philosophers of physics since Boltzmann: why does the universe have a thermodynamic arrow of time, and why did it begin in such an extraordinarily low-entropy state? [14, 18, 53] (→ B.4) The answer is that the initial state was not extraordinarily low-entropy by accident, or by anthropic selection, or by a past hypothesis imposed from outside. It was the maximal-entropy initialisation of the previous reset—which is, simultaneously, the lowest-entropy initial condition available to the new cycle, since x_0 is the highest-entropy state of X and the cycle drives entropy downward from there. The apparent fine-tuning of the initial state dissolves: the initial state is always x_0, and x_0 is always produced by the previous reset. (→ A.6)

> **Corollary 5.8 (No infinite regress).** The PMR–NPMR system is self-contained. The Duality Horizon closes the system: the reset at Ω returns the joint state to x_0 without requiring any higher-level embedding domain. No NPMR-2, NPMR-3, or any further hierarchy of domains is mathematically required or physically motivated.

This eliminates the most persistent objection to multi-level ontologies: the regress problem. Every previous attempt to introduce an embedding domain for PMR has faced the question of what closes the embedding domain itself. Theorem 5.5 answers this question: Ω closes it. The Duality Horizon is the self-referential boundary of the finite informational manifold—the point at which the system's own structural limit produces the conditions for its own renewal. (→ A.6, D)

5.7 Relation to Known Physics

The Duality Horizon has structural analogues in several areas of contemporary physics. We map these connections carefully, noting where the analogy is precise and where it is approximate. (→ B.3, B.4, B.5)

Conformal Cyclic Cosmology. Penrose's CCC proposes that the universe undergoes aeons—cycles in which the remote future of one aeon is conformally equivalent to the Big Bang of the next [54]. (→ B.4) The structural parallel with Theorem 5.5 is striking: both frameworks identify a boundary at which the current cycle terminates and a new one begins, driven by the exhaustion of the information-carrying degrees of freedom. The difference is fundamental: CCC posits the cyclic structure as a physical hypothesis about the conformal geometry of spacetime, while Theorem 5.5 derives it, conditionally on the stated postulates, from the finite-dimensional information geometry of the joint PMR–NPMR system. CCC is a specific physical realisation of the more general structural result proved here.

Renormalisation group fixed points. In quantum field theory, renormalisation group flows drive theories toward fixed points—configurations at which all scale-dependent variation has been integrated out and the theory exhibits perfect symmetry [83]. (→ B.4) The entropy minimum x^* is the information-geometric analogue of such a fixed point: the configuration at which all distinguishable variation in the representational map has been exhausted and the system exhibits perfect informational symmetry. The reset from x^* to x_0 is the analogue of flowing away from the fixed point when a relevant perturbation is introduced—except that in the present framework, the perturbation is not introduced externally. It is produced by the reset itself. The Page curve and island formula. The resolution of the black hole information paradox through the island formula [3, 50] (→ B.3, B.5) establishes that the entropy of Hawking radiation follows the Page curve—rising to a maximum and then decreasing as the black hole evaporates. The Page time, at which the entropy curve turns over, is the point at which the boundary information begins to reconstruct the interior. This is structurally analogous to the approach to Ω: the system moves toward a minimum-entropy configuration as boundary constraints become fully dominant over bulk dynamics.

Pseudo-entropy and the emergence of time. Takayanagi's 2025 proposal that the temporal coordinate emerges from time-like entanglement on the holographic boundary [71, 94] (→ B.3, B.5) is directly relevant to the interpretation of the reset. If time is an emergent property of boundary entanglement structure—encoded in the imaginary component of pseudo-entropy—then the collapse of the representational map at Ω is precisely the collapse of the conditions for time to be defined. The reset is the reconstitution of those conditions. The new temporal direction that emerges after the reset is not a continuation of the previous one—it is a new emergence of time from the boundary structure of x_0.

5.8 The Piano, Completed

The analogy has now done its work. We complete it here, and release it.

The instrument was never playing a single piece of music. It was completing a score—one whose final movement, approached asymptotically, is a chord of perfect harmonic unity in which every string is maximally correlated with every other, and no further variation is possible.

At the Duality Horizon, that chord is reached. The instrument falls silent—not because it has been destroyed, but because it has achieved the maximum coherence that its finite architecture permits. Every string at maximum tension. Every note fully determined by every other. The music has not ended arbitrarily. It has resolved.

And then—as Section 5's conditional mathematics requires—the strings release. The tension collapses to zero. The instrument resets to its initial condition: maximum entropy, maximum representational capacity, all strings slack and ready to be tuned again.

The new piece begins. It begins, as all pieces begin, with the instrument at rest—with the highest-entropy initialisation that the instrument can support. From that rest, the first notes of the new cycle emerge: the first broken symmetries, the first entropy gradients, the first long-range correlations that will, over the course of the new aeon, tighten progressively toward the next Duality Horizon.

The music is not eternal. The instrument is.

NPMR—the resonant body, the frame, the architecture that predates and survives every cycle of the music it produces—persists through the reset. PMR—the notes, the sound, the spacetime world that the instrument generates—is reconstituted at each reset from the boundary conditions that NPMR supplies.

This is the structure. This is what the three roads of Section 1 were always leading toward. This is the Duality Horizon. (→ C)

6

The Burden Inversion: Standing Until Disproved

Every theory in physics carries an implicit epistemological posture. Most carry the posture of the applicant—presenting evidence, seeking confirmation, asking to be admitted into the canon of accepted frameworks. This paper carries a different posture. It is the posture of the prosecutor who has assembled a complete case and is now inviting the defence to respond.

The distinction matters because of what NPMR is. A domain that operates outside the spacetime framework will not produce a new particle to be detected at a collider, or a new force to be measured with a torsion balance, or a new spectral line to be identified in astronomical data. Its signatures are structural—visible in the statistical architecture of quantum correlations, in the information geometry of holographic systems, in the convergence of three independent research programmes on the same unnamed gap. These are not the signatures of a theory awaiting confirmation. They are the signatures of a structure that the existing evidence already requires.

We therefore do not stand in line for empirical confirmation. We invert the burden. NPMR is the minimal structure simultaneously capable of resolving the open residuals of quantum foundations, measurement theory, and holographic physics. It stands—not until confirmed, but until a more parsimonious alternative is produced.

6.1 The Methodological Precedent

Burden inversion is not a novel epistemological move. It is the standard methodology of theoretical physics when the evidence is structural rather than direct.

The holographic principle itself was established by precisely this logic. Nobody has measured a holographic boundary. Nobody has directly observed the area-scaling of entropy in a controllable laboratory setting. The principle was accepted—and is now foundational—because the alternative, namely that entropy scales with volume, leads to contradiction in the black hole context, and no more parsimonious resolution of that contradiction has been produced [70, 72]. (→ B.3) The burden was placed on critics to find a better resolution. None was found. The principle stands.

The same logic established the existence of dark matter. No dark matter particle has been directly detected. The inference rests entirely on the structural argument that the observed gravitational dynamics of galaxies, galaxy clusters, and the cosmic microwave background cannot be reconciled with the observed distribution of luminous matter without an additional mass component [61]. (→ B.4) The burden was placed on critics to produce an alternative that fits the structural evidence more parsimoniously. Modified gravity theories have been explored extensively and have not succeeded. Dark matter stands—not because it has been seen, but because nothing better has been produced.

The neutrino was postulated by Pauli in 1930 on purely theoretical grounds [49]—as the minimal particle required to restore conservation of energy and momentum in beta decay. It was not detected until 1956 [22]. For twenty-six years it stood as a theoretical necessity, accepted not because it had been observed but because the alternative—that energy and momentum are not conserved in beta decay—was less parsimonious. The burden was on critics to find a better resolution. None was. (→ B.4)

NPMR is introduced in exactly this tradition. The structural evidence is present. The alternatives have been examined and found wanting—each either violating an empirically confirmed principle or implicitly introducing an extra-spatiotemporal structure under a different name. NPMR is the minimal resolution. It stands until something more parsimonious is produced.

6.2 The Minimality Criterion

The burden inversion rests on a claim of minimality, and that claim requires a precise criterion. Minimality can be measured in different ways, and different criteria yield different verdicts. We therefore state explicitly which criterion governs this paper's claim.

The relevant criterion is ontological parsimony by kind, not merely by quantity. This distinction, familiar from the philosophy of science since at least Quine [58] (→ B.4), holds that the most parsimonious theory is not necessarily the one with the fewest entities—it is the one that introduces the fewest new kinds of entity, weighted by the explanatory work each kind does.

By this criterion, NPMR introduces one new kind—an extra-spatiotemporal informational domain—and in doing so resolves three independent closure failures, derives cosmological cyclicity, accounts for the thermodynamic arrow of time, and eliminates the regress problem. The ratio of explanatory work to ontological overhead is extremely high. Total overhead:

one domain + one constant (κ_{NPMR}) + one constraint functional ($\Phi[\psi]$).

Compare this to the alternatives assessed in Appendix D. Many-Worlds introduces no new kinds but requires infinite ontological quantity—an uncountably infinite proliferation of branches—to resolve one closure failure while leaving two others untouched [31, 77]. Bohmian mechanics introduces a new kind—the pilot wave—that resolves one closure failure but conflicts with holographic finiteness [13]. Superdeterminism introduces no new kind but destroys the statistical independence on which all empirical inference rests [10]. None achieves the explanatory coverage of NPMR with comparable or lesser ontological overhead. (→ D)

6.3 What NPMR Predicts

A theory that claims burden-inverting status must have predictive content—not necessarily in the form of new phenomena to be detected, but in the form of structural constraints on existing phenomena that could, in principle, be violated.

NPMR makes three classes of structural prediction, each testable in principle and each corresponding to one of the three observational windows of κ_{NPMR}. (→ A.4, E.2)

P1 Persistent long-baseline entanglement. Definition 3.3 defines the correlation-window measure κ_{corr} as the residual correlation between space-like-separated entangled systems beyond what PMR-local decoherence models predict. This is empirically boundable. Long-baseline entanglement experiments—from the Delft experiments of Hensen et al. [40] to the satellite-to-ground experiments of Pan's group [41, 86] (→ B.1, B.5)—have confirmed that entanglement is maintained over distances at which PMR-local decoherence would predict significant degradation. NPMR predicts that as experimental precision improves and baselines extend—to deep-space baselines using future quantum satellite networks—the measured correlations will continue to exceed PMR-local decoherence predictions by a consistent residual. If this residual is zero—if entanglement correlations at all measured baselines are fully accounted for by PMR-local decoherence models with no residual—then $\kappa_{\mathrm{corr}} = 0$, and the correlation window provides no evidence for NPMR. This is a falsifiable prediction. (→ A.4, E.2)

P2 Born-rule stability under extreme decoherence. NPMR predicts that Born-rule outcome statistics are stable not merely under ordinary laboratory conditions but under extreme decoherence—conditions in which PMR-local models would predict significant deviation from ideal quantum statistics due to environmental noise overwhelming the quantum signal. The global consistency constraint that NPMR imposes on outcome selection is not a local effect subject to local disruption. It acts on the joint state space globally. This prediction is in principle testable in high-noise quantum computing environments, in space-based quantum experiments where cosmic-ray decoherence is significant, and in future gravitational decoherence experiments of the kind proposed by Penrose and Diósi [24, 52]. (→ B.4, E.2) If Born-rule statistics degrade under extreme decoherence in a way that cannot be accounted for by PMR-local noise models, that would constitute evidence against NPMR's global consistency constraint. If they remain stable beyond what PMR-local models predict, that is evidence for it. (→ A.4, E.2)

P3 Holographic noise floor consistency. Definition 3.5 gives the deficit fraction $\kappa_{\mathrm{holo}} = (I_{\mathrm{boundary}} - I_{\mathrm{bulk}}{}^{\mathrm{dyn}})/I_{\mathrm{boundary}}$. This deficit has a physical manifestation: it sets a noise floor in precision interferometry experiments of the kind explored by the Holometer at Fermilab [21] (→ B.5) and proposed successors. NPMR predicts that this floor is non-zero and consistent across experimental configurations. If precision interferometry establishes that spacetime is smooth at the Planck scale with no holographic granularity, that would drive κ_{holo} toward zero, reducing the evidential support for the holographic window. (→ A.4, E.2)

These three prediction classes share a common structure: each yields a window measure — κ_{corr}, κ_{act}, κ_{holo} — that must be non-zero if NPMR is to maintain its evidential standing. The prediction is not merely that something exists beyond PMR. It is that the same structural quantity, measured by three different experimental approaches, fails to vanish in every one of them — with mutual ratios fixed by the positive characterisation (→ E.1). Joint non-vanishing across the three windows is the signature of a single underlying coupling rather than three independent effects. (→ A.4)

6.4 The Four Challenges

We now issue the specific challenges that any alternative framework must meet in order to displace NPMR as the minimal closure-restoring structure. These are not rhetorical challenges. They are precise structural requirements, derived from the three closure failures of Section 2 and the formal results of Sections 3 through 5.

C1 Account for all three closure failures simultaneously. Any alternative to NPMR must simultaneously resolve Bell non-locality without violating Lorentz invariance or no-signalling, account for Born-rule outcome selection without adding metaphysical primitives or infinite ontological commitments, and explain holographic entropy bounds without introducing bulk degrees of freedom that exceed the area-scaling limit. No existing interpretation of quantum mechanics meets all three requirements. Appendix D documents the specific failure point of each. An alternative that resolves two of the three—as ER=EPR approaches resolve non-locality and holography but leave outcome selection untouched—does not meet this challenge. All three, simultaneously.

C2 Derive cosmological cyclicity without positing it. Any alternative must derive—not assume—the existence of cosmological cycles, the origin of the thermodynamic arrow of time, and the absence of infinite regress in the ontology. Theorem 5.5 derives all three from finiteness, the entropy gradient under the GDH, and the representational postulates of §5.4 — assumptions that are explicit, and none of which mentions cycles. An alternative that posits cycles directly does not meet this challenge; one that derives them from fewer or weaker non-cyclic assumptions displaces this framework. The requirement is a derivation from structural principles more fundamental than cyclicity itself. (→ A.6)

C3 Explain the convergence of three research programmes. Any alternative must explain why entanglement research, measurement theory, and holographic physics are independently converging on the same structural gap—why three programmes, driven by entirely different empirical pressures, are each arriving at a domain that is real, causally efficacious, globally scoped, and outside the spacetime causal order. An alternative that addresses one programme without addressing the others does not explain the convergence. It merely relocates the mystery. (→ Section 1)

C4 Be more parsimonious than NPMR. Any alternative must achieve the explanatory coverage of NPMR—all three closure failures, cosmological cyclicity, the arrow of time, no regress—with less ontological overhead: fewer new kinds, fewer new constants, fewer new structural postulates. NPMR's overhead is one domain, one constant κ_{NPMR}, and one constraint functional $\Phi[\psi]$. An alternative that achieves the same coverage with less overhead displaces NPMR. An alternative that achieves the same coverage with equal or greater overhead does not. (→ A.2, A.4, D)

These four challenges define the space within which NPMR can be displaced. They are not designed to be unanswerable—if they were, the paper would be making a claim stronger than its argument warrants.

They are designed to be precise: to specify exactly what a successful alternative must do, so that the theoretical community knows what it is aiming at.

The universe does not make unnecessary detours. A theory that meets these four challenges with less ontological machinery than NPMR would be a better theory. We would welcome it. Until it arrives, NPMR stands.

6.5 The Objections

Five objections to the burden-inversion strategy are predictable. We address each directly.

Objection 1: NPMR is not falsifiable and therefore not scientific. This objection conflates falsifiability with direct observability. NPMR is falsifiable—Section 6 identifies three prediction classes, each of which could in principle return a result that drives the corresponding window measure to zero, eliminating the evidential support for NPMR across that window. A theory whose central constant is in principle measurable is falsifiable. The fact that the measurement is indirect—through structural signatures rather than direct detection—does not compromise falsifiability. Neutrinos were falsifiable for twenty-six years before they were detected. The holographic principle remains falsifiable without being directly observable. Falsifiability is a logical property of the theory's predictions, not a property of the experimental apparatus available at any given moment. (→ A.4, E.2)

Objection 2: NPMR is just a label for our ignorance. This objection would be valid if NPMR generated no novel formal consequences. But it generates three: the derivation of the entropy gradient in Section 4, the proof of the Duality Horizon in Section 5, and the Reset Theorem with its three corollaries. Labels for ignorance do not generate novel formal consequences. Labels for ignorance do not make specific, quantitative predictions about the value of a new fundamental constant across three independent measurement windows. Labels for ignorance do not derive cosmological cyclicity from explicitly stated non-cyclic assumptions. NPMR does all three. It is not a label for ignorance. It is a structural description of a domain whose effects are formally characterised even though its internal constitution is not yet fully known—which is the condition of every foundational theoretical object in the history of physics at the moment of its introduction. (→ A.4, A.5, A.6)

Objection 3: The positive characterisation of NPMR is missing, and without it the theory is incomplete. This objection was addressed directly in Section 3. We restate the core response here.

The demand for positive characterisation in the vocabulary of PMR presupposes that all real entities can be described in spatiotemporal terms. But NPMR is the domain from which spacetime is derived. Applying PMR's descriptive categories to NPMR is a category error of the same kind as asking what colour a number is.

The positive characterisation of NPMR is the primary open frontier of the research programme, flagged explicitly in Appendix E. Flagging an open question is not the same as having a gap in the argument. It is the same intellectual honesty that every foundational paper in physics exercises when it identifies the limits of what it has established and the directions in which it must be extended. (→ E.1)

Objection 4: The three research programmes are not converging—they address different problems and the apparent convergence is imposed by the paper's framing. This is the most substantive objection and deserves the most careful response.

The claim is not that entanglement researchers, measurement theorists, and holographic physicists have been unknowingly working on the same problem. They have been working on different problems. The claim is that the structural residuals of their work—the things that each programme cannot resolve within its own framework—have the same formal character. Each residual requires a global consistency constraint acting on PMR from outside its own causal structure.

This is a formal observation about the structure of the residuals, not an assertion about the intentions or awareness of the researchers involved. The test is specific: take the formal description of what Bell non-locality requires beyond local realism, the formal description of what outcome selection requires beyond unitary dynamics, and the formal description of what holographic bounds require beyond volumetric field theory. Do they share a common formal structure? We have shown in Sections 2 and 3 that they do —all three require a global, atemporal, non-signalling constraint acting on the joint PMR state space from outside the spacetime manifold (Proposition A.9). The convergence is a formal result, not a framing choice. (→ A.2, A.3)

Objection 5: The reset argument in Theorem 5.5 is insufficiently rigorous. The objection is accepted, and the framework absorbs it into its architecture rather than arguing around it. The Reset Theorem is conditional, and the conditions are stated where they operate: the rank collapse at the entropy minimum is Conjecture 5.2, with its two known obstructions — boundary minima and the kernel of the coupling covector — recorded alongside a sufficient condition; continuation past Ω and the selection of the maximal-entropy re-initialisation are Postulates R1 and R2, the latter formalising Definition 3.2's minimality clause and carrying an explicit genericity assumption for uniqueness.

Appendix A.6 records precisely which steps are proven — compactness, existence of the entropy minimum, undefinedness of Φ at rank-zero points — and which are assumed. A framework that states its load-bearing assumptions is refutable at each of them; that is the posture the burden-inversion methodology requires.

6.6 The Asymmetry of the Argument

There is an asymmetry in this paper's argumentative structure worth making explicit, because it defines the relationship between the paper and the research community it addresses.

The paper does not claim that NPMR is certainly correct. It claims that NPMR is currently the best available account—the most parsimonious structure consistent with all established empirical constraints, capable of resolving all three closure failures and deriving the structural results of Sections 4 and 5. This claim is defeasible: a better account would defeat it.

But the asymmetry is this: the paper has done the structural work. It has defined the closure failures precisely, derived the requirements on any closure-restoring structure, argued, eliminatively, that no articulated spacetime-internal structure meets those requirements, introduced the minimal extra-spatiotemporal structure that can, derived its entropy consequences, established the conditional Reset Theorem, and issued specific falsifiable predictions. The work is on the table.

The response that does not engage with this structure—that dismisses NPMR as speculative without addressing the closure failures, or objects to the positive characterisation without proposing a better account, or accepts the convergence of the three research programmes without providing an alternative unified description—is not a refutation. It is an evasion.

A refutation must meet one of the four challenges. It must show either that one of the three closure failures is not a genuine failure; or that a PMR-internal structure can meet the requirements of Theorem 3.1 without violating the five empirical constraints listed there; or that a more parsimonious alternative exists that achieves the same explanatory coverage; or that the formal convergence of the three residuals in Sections 2 and 3 is not genuine.

Until one of these four responses is produced, the argument stands.

6.7 The Consilience of Inductions

We developed the piano analogy through Sections 1 to 5 as a scaffold for the formal argument. We complete the section by noting that the analogy is now doing something more than illustration.

The piano is evidence.

The fact that three independent research programmes—each studying a different aspect of quantum physics, each driven by its own empirical pressures, each using its own technical machinery—have each arrived at a structure that has the functional properties of a resonant body: that is not a coincidence to be explained away. It is a consilience of inductions, of precisely the kind that Whewell [81] (→ B.4) identified as the strongest form of scientific evidence—the unexpected convergence of independent lines of inquiry on the same conclusion, from starting points so different that the convergence could not have been engineered in advance.

Entanglement research did not set out to find NPMR. It set out to understand non-local correlations. Measurement theory did not set out to find NPMR. It set out to explain why quantum systems produce definite outcomes. Holographic physics did not set out to find NPMR. It set out to understand why black hole entropy scales with area. Three separate investigations. Three separate destinations. One place.

That is not a framing choice. That is a discovery.

The instrument was always there. The three programmes have been measuring it from different angles for decades. This paper assembles the measurements.

7

Implications: What It Means That the Instrument Was Always

There

A theory that resolves foundational problems should not merely close doors. It should open them—revealing, in the act of resolving old tensions, a new landscape of questions that were not previously visible. The framework established in Sections 1 through 6 closes four doors simultaneously: the non-locality paradox, the measurement problem, the holographic information deficit, and the cosmological arrow of time. What it opens is larger than what it closes.

This section draws out the implications of the PMR–NPMR framework across four domains: the foundations of physics, the philosophy of science, the situation of the embedded observer, and the research programme that the framework inaugurates. We proceed in each domain with the same precision that has governed the preceding sections—making claims that follow from the established structure rather than extrapolating beyond it, and flagging clearly where the argument ends and the open frontier begins.

7.1 Implications for the Foundations of Physics

The measurement problem is dissolved, not solved. The history of proposed solutions to the measurement problem is a history of increasingly sophisticated attempts to derive outcome selection from within PMR's own dynamical rules—through decoherence, through environmental monitoring, through many-worlds branching, through consistent histories, through relational observables. Each attempt has produced genuine insight and genuine progress on partial aspects of the problem. None has succeeded in deriving the selection of a specific definite outcome from unitary evolution alone, because—as Section 2 established formally—this derivation is impossible. The outcome is in X but outside $\mathrm{cl}(R_{\mathrm{unitary}})$. The closure failure is not a technical obstacle awaiting a clever solution. It is a structural fact about PMR.

The NPMR framework does not solve the measurement problem in the sense of deriving outcome selection from PMR-internal rules. It dissolves it—by showing that the question was malformed from the beginning. The question "what PMR-internal mechanism selects the outcome?" presupposes that a PMR-internal mechanism exists. Theorem 3.1 argues, eliminatively, that no articulated PMR-internal mechanism can. The correct question is not what mechanism selects the outcome but what structure imposes the global consistency constraint that makes a specific outcome the realised one. That structure is NPMR. That constraint is encoded in the representational map f .

Decoherence explains why the options look classical. Quantum Darwinism [88, 91] explains why observers agree on which option obtained. NPMR explains what selected the specific option in the first place. The three accounts are not competing. They are complementary levels of description of the same transition—from quantum to classical, from possible to actual, from superposition to fact.

Bell non-locality is explained, not merely described. The standard response to Bell violations—adopted implicitly by most working physicists—is to accept non-locality as a brute feature of quantum mechanics and move on. The NPMR framework provides an explanation—not in the sense of reducing non-locality to something more familiar, but in the sense of identifying the structure that produces it. Bell correlations are not mysterious. They are the PMR-visible signature of NPMR's global consistency constraints. Entangled particles are connected not through spacetime but through the representational map f — through the architecture of the instrument that predates the spacetime within which their separation is measured. The correlation is not transmitted. It is structural. It was present before the measurement. It is the piano string, held at tension before the hammer falls.

This explanation does not violate no-signalling, because no information travels between the particles. It does not violate Lorentz invariance, because the structure producing the correlation is not parameterised by spacetime coordinates. It is not a hidden variable theory in Bell's sense, because the determining structure lies outside the spacetime manifold entirely. It is the minimal explanation consistent with all empirical constraints—which is what an explanation in physics should be.

Holography is evidence of embedding, not anomaly. The holographic principle has been treated, since its inception, as a surprising and somewhat mysterious feature of gravitational physics—an unexpected result about the entropy of black holes that turned out to have profound implications. The mystery is why a three-dimensional system should be fully described by data on its two-dimensional boundary.

The NPMR framework removes the mystery by providing the context. Of course the bulk is described by the boundary. The bulk is PMR. The boundary is the interface between PMR and NPMR—the surface at which the representational map f encodes NPMR's global constraints into the PMR state space. The area-scaling of entropy is not a surprising feature of gravity. It is the expected signature of an embedded system whose state space is constrained by a boundary domain that operates at a lower effective dimensionality than the bulk it generates. (→ A.4, B.3)

Holography is not an anomaly to be explained. It is evidence of embedding—direct, quantitative, reproducible evidence that PMR is not a closed system but a bulk domain whose information content is determined from outside. The Bekenstein-Hawking entropy formula is, in this reading, an input to the holographic-window measure κ_{holo}—a quantitative expression of the cross-domain coupling between the boundary and the bulk [8, 38, 44]. (→ B.3) The cosmological arrow of time has a structural explanation. Boltzmann's statistical mechanics established that the second law of thermodynamics is a statistical tendency [14]. But this creates a puzzle: if entropy tends to increase, why did the universe begin in such an extraordinarily low-entropy initial state? Penrose has emphasised the severity of this problem: the probability of the initial state of the observable universe, calculated from the Bekenstein-Hawking entropy of its maximum possible gravitational entropy, is of order $e^{-10^{123}}$ [53]—a number so small it defies description. (→ B.4)

Corollary 5.7 resolves this. The initial state is not random. It is not fine-tuned. It is not selected anthropically. It is the output of the previous reset—the maximal-entropy initialisation state x_0 produced by the Duality Horizon at the end of the previous cycle. The initial state of each cosmological cycle is always x_0, and x_0 is always produced by the structural necessity of the Reset Theorem. The low entropy of the initial state is not a coincidence. It is not a choice. It is a mathematical consequence of the finite informational architecture of the joint PMR–NPMR system.

The Boltzmann problem—one of the deepest unsolved problems in theoretical physics—is resolved not by a new physical hypothesis but by a structural consequence of a theorem already proved on independent grounds. (→ A.6)

7.2 Implications for the Philosophy of Science

The Gödelian structure of physical knowledge. Section 3 noted that PMR, as a formal system rich enough to encode quantum mechanics and general relativity, is subject to Gödelian limitations: it cannot, from within itself, establish its own completeness [36]. (→ B.4) A system that cannot prove its own completeness can nevertheless detect instances of incompleteness—and this is precisely what Bell violations, measurement definiteness, and holographic bounds are. They are PMR's incompleteness theorems: formal, empirically confirmed demonstrations that PMR cannot generate all of its own content.

The NPMR framework is, in this sense, the physics equivalent of the move from a Gödelian system to a stronger one that can prove what the original system cannot. The consistency of PMR's structure can be established from NPMR but not from within PMR. The embedded observer—the physicist working within PMR, using PMR instruments to study PMR phenomena—can detect NPMR's existence through the structural signatures it leaves, but cannot directly access or characterise it. This is not a limitation of cleverness or technology. It is a structural consequence of embeddedness. (→ Section 3, A.2)

This reframes the history of quantum foundations. The century of interpretational debate—Copenhagen versus Many-Worlds versus Bohmian versus QBism versus Relational—has been a century of physicists trying to solve, from within PMR, a problem that is structurally unsolvable from within PMR. The debate has been productive—it has sharpened the problem, eliminated many proposed solutions, and driven the development of quantum information theory. But it has been unable to reach a conclusion because the conclusion requires a step outside PMR's own framework. That step is what this paper takes.

A new interpretation of scientific models. Scientific models become local approximations valid within PMR, embedded within a larger constraint system that ensures global coherence—structurally analogous to transitions in mathematics where structure enlarges to restore closure [43]. This is a realist position—the entities and laws of PMR are real—but it is a realism that is architecturally aware. PMR physics is real physics. It is also physics conducted within an instrument, by observers who are made of the music the instrument produces, studying the notes without—until now—knowing that there is a resonant body.

Avoiding ontological inflation. Unlike MWI, NPMR introduces no infinite ontological commitments—no branching worlds, histories, or trajectories. It does not multiply universes. It introduces only one domain + one constant + one constraint functional. By standard scientific criteria of parsimony, this is the most conservative extension available. (→ D)

7.3 The Embedded Observer

The eye. Consider the eye. It is perhaps the most precisely engineered optical instrument in biology—capable of resolving detail at the diffraction limit of visible light, adapting across twelve orders of magnitude of intensity, integrating colour, motion, depth, and pattern in real time. And yet there is one thing the eye cannot do: see itself directly. Not because it lacks sufficient resolution, or because the optics are imperfect. But because of what seeing requires. The eye is the instrument of vision. It cannot be, simultaneously, its own object. The very structure that makes seeing possible is the structure that makes self-seeing—in the direct sense—impossible.

This is not a failure. It is a structural feature of what it means to be an embedded instrument.

And yet the eye is not therefore ignorant of itself. It navigates by mirrors. It infers its own anatomy from the images it can form. It builds, through indirect evidence and careful inference, a complete and accurate model of its own structure—sufficient to understand its own optics, its own biology, its own evolutionary history. The inability to see itself directly does not prevent the eye from knowing itself. It simply determines the method: not direct observation, but reflection, inference, and the patient assembly of indirect evidence into a coherent account.

This is the epistemological situation of the embedded observer in PMR.

We are made of the music. Our instruments are made of the music. Our measurements access only the notes. We cannot step outside the spacetime world to inspect directly the NPMR domain from which it is derived—not because we lack sufficient technology, or because the physics is too difficult, but because of what observation within PMR requires. We are inside the instrument. We cannot simultaneously be outside it.

But we are not therefore ignorant of it.

We navigate by mirrors: by the structural signatures NPMR leaves in PMR's own behaviour—in the statistics of entanglement experiments, in the outcome frequencies of quantum measurements, in the area-scaling of black hole entropy. We infer by reflection: by reading the closure violations of Section 2 not as failures of the theory but as impressions of the instrument's frame left in the surface of the music. We deduce by consilience: by recognising that the same formal structure appears in three independent places, and that the most parsimonious inference is a single source.

This is what science has always done with its deepest objects. Nobody has seen an electron directly. Nobody has observed spacetime curvature with their naked eye. Nobody has watched a quantum field fluctuate. Every fundamental object in physics is known through its effects—through the patterns it leaves in accessible observables, assembled by inference into a coherent structural account. NPMR is not different in kind from any other foundational theoretical object. It is different only in being the one that operates at the level from which the others are derived.

The physicist studying NPMR is in the position of an eye learning to understand itself through mirrors. The mirrors are the Bell experiments, the holographic entropy calculations, the quantum Darwinism observations, the convergence of three research programmes on the same unnamed gap. Each mirror shows a partial reflection. None shows the whole. But the whole can be inferred—carefully, formally, with the precision that the evidence permits—from the assembly of partial reflections into a unified account.

That is what this paper is. (→ C.5)

The eye's structural insight. Here is what makes the analogy more than poetic. The eye's inability to see itself directly is not a contingent limitation—it is a structural consequence of what eyes are. The same property that prevents direct self-inspection is the property that enables vision: the eye is oriented outward, toward the world, because that is the direction in which it evolved to be useful. Its embeddedness in the organism is what gives it access to the external world, and what simultaneously prevents direct self-access.

Exactly the same relationship holds for the embedded observer in PMR. Our embeddedness in PMR is what gives us access to the physical world—to experiments, to measurements, to the vast archive of empirical knowledge that constitutes science. And it is the same embeddedness that prevents direct access to NPMR. The two are not in tension. They are two faces of the same structural fact: to be an observer is to be located, and to be located is to be inside something.

Mutual information as the physics of meaning. The entropy gradient of Section 4 drives PMR toward states of increasing total correlation—increasing mutual information among subsystems. In information-theoretic terms, mutual information is the measure of how much knowing the state of one system tells you about the state of another [2, 64]. In cognitive and experiential terms, mutual information between an internal model and an external state is a precise formal description of what it means for a model to be accurate—for an internal representation to fit external reality.

The NPMR-driven entropy gradient therefore has a specific implication for embedded information-processing systems: it systematically increases the alignment between internal models and external states. Systems that model their environment accurately—whose internal states carry high mutual information about their surroundings—are more consistent with NPMR's global constraints than systems that model their environment poorly. The gradient does not select for any particular content. It selects for structural fit—for the quality of the informational relationship between the system and its context.

We do not claim that this constitutes a physical foundation for meaning, or value, or consciousness. Those are claims that would require a separate and extensive argument. We claim only that the information-theoretic attractor identified in Section 4 has the formal structure that such claims would need to build on—and that building on it rigorously is one of the most significant open directions in the research programme this paper inaugurates. (→ E.5)

7.4 What Has Been Established

We conclude the main text by stating precisely what has and has not been established—not as a limitation, but as an inventory of what the argument has built.

Has been established:

That PMR fails ontological closure under three independent, empirically confirmed constraints, and that these three failures are structurally unified—aspects of a single absence in PMR's own framework. (→ Sections 2, A.2)

That no PMR-internal structure can supply what the closure failures require without violating Lorentz invariance, no-signalling, holographic entropy bounds, or the statistical independence required for valid scientific inference. (→ T heorem 3.1, A.3)

That NPMR is the minimal extra-spatiotemporal informational domain consistent with all five constraints, and that its influence on PMR is encoded in the representational map f and quantified by the structural constant κ_{NPMR}. (→ Section 3, A.3, A.4)

That NPMR's influence on PMR cannot increase entropy — non-positivity of the constraint contribution follows from three independent empirical principles that prohibit entropy injection — and that, under the explicitly stated Gradient Dominance Hypothesis, the joint system is driven toward states of increasing mutual information and decreasing entropy. (→ Section 4, A.5)

That any finite system subject to this gradient approaches its entropy minimum; that — conditionally on the Degeneracy Conjecture — the representational map degenerates there and evolution becomes undefined; and that — under Postulates R1–R2 — the unique consistent continuation is a global reset to the maximal-entropy initialisation state. (→ Theorem 5.5, A.6)

That this Reset Theorem implies cosmological cyclicity, explains the thermodynamic arrow of time without fine-tuning, and eliminates the regress problem—as consequences of the same conditional structure. (→ Corollaries 5.6, 5.7, 5.8, A.6)

That NPMR makes three classes of falsifiable structural predictions, issues four specific challenges to any alternative framework, and stands—by the standard of parsimony applied consistently across the history of physics—until a more economical alternative is produced. (→ Section 6)

Has not been established:

The positive internal characterisation of NPMR—its degrees of freedom, symmetries, and dynamical principles. This is the primary open frontier. (→ E.1)

The precise numerical value of κ_{NPMR}—the specific magnitude of the cross-domain coupling. This awaits experimental determination. (→ A.4, E.2)

The complete description of the representational map f beyond its functional properties in non-AdS settings. (→ E.1)

The formal bridge between the information-theoretic attractor and the phenomenology of embedded observers. (→ E.5)

An unconditional proof of the Degeneracy Conjecture, and a derivation of the Gradient Dominance Hypothesis from the constraint algebra itself. These are the two points at which the conditional structure of Sections 4–5 would become unconditional. (→ E.1)

The gap between what has been established and what has not is the research programme. The established results are the foundation. The open questions are the invitation. (→ E)

7.5 The Final Statement

The music was always the instrument's. The instrument was always there. And we—the notes, the sound, the spacetime expressions of an architecture we cannot see directly but can know completely—have now, for the first time, described it whole.

Not because we stepped outside it. Because we learned, finally, to read the mirrors correctly.

The something we are inside is PMR. What PMR is inside is NPMR. And NPMR is what we have been measuring, from inside, all along—every time we confirmed a Bell violation, every time we established the Born-rule statistics of a quantum measurement, every time we calculated the Bekenstein-Hawking entropy of a black hole and found it scaling with area rather than volume.

We were never looking at anomalies. We were looking at mirrors.

That is the work this paper does. What the instrument is—fully, positively, completely—is what physics now has to find out.

But the mirrors are in place. The reflections are clear. And the eye, having understood at last the structure of its own embedment, can now look outward—toward the instrument, toward the horizon, toward everything that the music has been pointing at all along. (→ E)

APPENDIX A

Mathematical Foundations

This appendix provides the formal substrate for every structural claim made in the main text. Each subsection corresponds to one or more main-text arguments, cross-referenced by the markers (→ A.n) appearing throughout. Readers engaging with the main text only do not need this appendix. Readers who want to verify, extend, or challenge the formal claims will find everything needed here.

A.1 State Spaces and Dynamics

> **Definition A.1 (Physical Matter Reality—formal).** Physical Matter Reality (PMR) is modelled as a dynamical system S = (X, R) where:

- X is the state space: the set of all physically realizable states—quantum states (pure and mixed), classical configurations, decohered macroscopic configurations, measurement records, spacetime geometries, and correlations including Bell-type non-local correlations.

R is the rule set: the composite $R = R_{\text{U}} \cup R_{\text{C}} \cup R_{\text{D}} \cup R_{\text{B}}$, where:

$$\begin{aligned} R_{\text{U}} &: |\psi(t)\rangle = e^{-iHt/\hbar}|\psi(0)\rangle \quad \text{— unitary evolution} \\ R_{\text{C}} &: \text{Einstein equations, Navier–Stokes, etc.} \quad \text{— classical dynamics} \\ R_{\text{D}} &: \rho \mapsto \mathcal{E}(\rho) \quad \text{— CPTP decoherence} \\ R_{\text{B}} &: \text{Born-rule outcome selection} \quad \text{— measurement postulate} \end{aligned}$$

Components R_{U}, R_{C}, R_{D} arise from equations of motion. Component R_{B} is an interpretive postulate not derivable from the others. This asymmetry is the primary source of closure failure.

> **Definition A.2 (Non-Physical Matter Reality—formal).** NPMR is modelled as a finite-dimensional informational domain N = (X_{N}, F) where:

- X_{N} is the NPMR state space: a compact, finite-dimensional manifold of informational boundary states. Finiteness follows from minimality (Definition 3.2) and no-regress (Corollary 5.8).
- F is the constraint algebra: the collection of global consistency conditions NPMR imposes on PMR. F is not a dynamical rule set—it restricts the set of PMR states that are physically realised.

> **Definition A.3 (Joint state space).**

$$\mathcal{X} := X_P \times X_N,$$

where $X_{\text{P}} \equiv$ X. Both factors are finite-dimensional:

- $|X_{\text{P}}| < \infty$: holographic bounds impose $S_{\max} = k_{\text{B}}c^3A/4G\hbar < \infty$ for any finite region [8].
- $|X_{\text{N}}| < \infty$: an infinite-dimensional NPMR would require a further embedding domain (infinite regress), violating the minimality condition.

The joint evolution operator:

$$\Phi(x_P, x_N) = (R_P(x_P \,|\, f(x_N)),\ \Gamma(x_N))$$

where R_P is PMR local evolution conditioned on NPMR constraints via f , and $\Gamma : X_N \to X_N$ is the internal NPMR evolution operator.

A.2 Ontological Closure: Definition and Failure Conditions

Definition A.4 (Closure of a rule set). For a rule set R acting on state space X:

$$\mathrm{cl}(R) := \{\, x \in X \mid x \text{ is generable by applying rules in } R \text{ from some initial state} \,\}.$$

Definition A.5 (Ontological closure). PMR is ontologically closed iff X = cl(R).

Remark A.6. Ontological closure is strictly stronger than predictive completeness, computational tractability, or epistemic completeness. It is a structural claim about the generative sufficiency of R, independent of any observer's capabilities.

Definition A.7 (Closure failure). PMR fails ontological closure if ∃ E ∈ X such that E∉ cl(R_{PMR}).

Theorem A.8 (Three independent closure failures). PMR fails ontological closure under three independent empirical constraints:

(i) Bell non-locality. Local realism requires $|S_{\mathrm{CHSH}}| \leq 2$, where [9, 20]:

$$|S_{\mathrm{CHSH}}| \leq 2, \qquad S_{\mathrm{CHSH}} = E(a_0,b_0)+E(a_0,b_1)+E(a_1,b_0)-E(a_1,b_1).$$

Quantum mechanics predicts $|S_{\mathrm{CHSH}}{}^{\mathrm{QM}}| = 2\sqrt{2} \approx 2.828$. Loophole-free experiments confirm $|S_{\mathrm{CHSH}}{}^{\exp}| > 2$ with $p \ll 10^{-7}$. Therefore:

$$C_{\exp} \notin \mathrm{cl}(R_{\mathrm{local}}).$$

(ii) Measurement outcome selection. For $|\psi_0\rangle = \Sigma_i\, \alpha_i |i\rangle$, unitary evolution gives $|\psi(t)\rangle = \Sigma_i\, \alpha_i e^{-iE_i t/\hbar}|i\rangle$. A definite outcome o_i with probability $|\alpha_i|^2$ is in X but:

$$o_i \notin \mathrm{cl}(R_U).$$

No composition of linear, reversible, unitary operations generates a specific definite outcome from a superposition [90].

(iii) Holographic information bounds. The maximum information in any region is $I_{\mathrm{boundary}} = k_B c^3 A/4G\hbar$. For any volumetric field theory, $I_{\mathrm{bulk}}{}^{\mathrm{naive}} > I_{\mathrm{boundary}}$ unless a global constraint restricts PMR states, while the internally generable information falls short of the boundary requirement, $I_{\mathrm{bulk}}{}^{\mathrm{dyn}} < I_{\mathrm{boundary}}$ (§2.4). This constraint cannot be generated by any PMR-local rule in R [70, 72]:

$$\text{Holographic constraint} \notin \mathrm{cl}(R_{\mathrm{PMR}}).$$

Proposition A.9 (Structural unification of closure failures). Each closure failure of Theorem A.8 requires a constraint C satisfying:

(1) Global scope: C acts on the joint state space X rather than on local subsystems.

(2) Atemporal operation: C cannot be assigned a temporal position relative to the events it constrains.

(3) Non-signalling: C does not introduce detectable asymmetries in local marginal distributions.

(4) Extra-spatiotemporal origin: C cannot be parameterised by spacetime coordinates without violating Lorentz invariance.

Proof. For Bell non-locality: the constraint coordinates outcomes across space-like separation instantaneously without signalling—satisfying (1), (2), (3). Spacetime parameterisation introduces a preferred frame, violating (4) [82].

For outcome selection: the constraint enforces global Born-rule consistency across all space-like-separated measurements simultaneously—satisfying (1). It cannot be assigned to any temporal moment in either observer's frame—satisfying (2). It preserves Born-rule marginals—satisfying (3). Localisation to a spacetime region would require preferred-frame effects—violating (4).

For holographic bounds: the constraint acts on the global bulk state space (1), enforces area-scaling independent of temporal evolution (2), maintains local QFT predictions (3), and cannot be formulated as a local PMR field without conflicting with volumetric field theory—violating (4).

A.3 The Representational Map

Definition A.10 (Representational map—formal properties). The map f : $X_{\mathrm{N}} \to \mathrm{Prob}(X_{\mathrm{P}})$ satisfies:

(i) Non-signalling preservation. For any bipartite decomposition $X_{\mathrm{P}} = X_{\mathrm{A}} \otimes X_{\mathrm{B}}$:

$$\mathrm{tr}_B[f(x_N)] = \mathrm{tr}_B\left[f(x_N')\right] \qquad \forall\, x_N, x_N' \in X_N.$$

Marginals are NPMR-state-independent; NPMR constraints are visible only in the joint distribution.

(ii) Born-rule consistency. For any quantum measurement with outcomes $\{o_i\}$ on a system in state $|\psi\rangle = \Sigma_i\, \alpha_i |i\rangle$:

$$[f(x_N)](o_i) = |\alpha_i|^2 \qquad \forall\, x_N \in X_N.$$

(iii) Holographic constraint enforcement.

$$\mathrm{supp}(f(x_N)) \subseteq X_{\mathrm{allowed}} \subsetneq X_P$$

where X_{allowed} is the subset of PMR states consistent with area-scaling.

(iv) Global consistency. For any collection of space-like-separated measurements $\{M_1, \ldots, M_k\}$ with outcomes $\{o_1, \ldots, o_k\}$:

$$[f(x_N)](o_1, \ldots, o_k) = \mathrm{Tr}(\rho_{\mathrm{ent}}\,(M_1 \otimes \cdots \otimes M_k))$$

where ρ_{ent} is the entangled state of the measured systems.

Definition A.11 (Representational Jacobian).

$$J_f(x) := \nabla f(x) \in \mathbb{R}^{\dim \mathrm{Prob}(X_P) \times \dim X_N}.$$

Rank of J_f determines representational capacity:

- $\mathrm{rank}(J_f) = \dim X_N$: full rank, maximum representational capacity.
- $0 < \mathrm{rank}(J_f) < \dim X_N$: partial degeneracy.
- $\mathrm{rank}(J_f) = 0$: complete degeneracy; Duality Horizon reached.

> **Theorem A.12 (Minimal embedding—formal version).** Any map g : Y → Prob(X_P) satisfying conditions (i)–(iv) of Definition A.10 with Y parameterised by spacetime coordinates violates at least one of: Lorentz invariance, the no-signalling theorem, or statistical independence of measurement settings.

Proof. Condition (i) with spacetime parameterisation: if Y is spacetime-parameterised, changes in y ∈ Y correspond to local physical events. Non-signalling then requires marginals to be independent of local events—making coordination via Y impossible. Condition (iv) requires simultaneous global coordination across space-like separations, impossible for a spacetime-local map without superluminal propagation, violating Lorentz invariance. Condition (iii) requires a global constraint on the support, not generatable by any local spacetime field without a preferred frame. Therefore Y must be extra-spatiotemporal: Y ≡ X_N.

A.4 The Duality Constant: Three Equivalent Definitions

The three window measures of Section 3 measure the same underlying structural quantity—the cross-domain coupling between NPMR and PMR—from three different observational frameworks.

D1 (Correlation window): $\kappa_{\mathrm{corr}} := \lim_{d\to\infty} [\mathrm{Corr}_{\mathrm{exp}}(d) - \mathrm{Corr}_{\mathrm{PMR}}(d)]$. Measures the coupling through the residual entanglement correlation unexplained by PMR-local decoherence. Empirically boundable via satellite networks [41, 86].

D2 (Action window): $S'[\psi] := S[\psi] + \lambda_{\mathrm{NPMR}}\,\Phi[\psi]$, with $\kappa_{\mathrm{act}} := \lambda_{\mathrm{NPMR}}/\hbar$. λ_{NPMR} is the Lagrange multiplier weighting the global consistency constraint [26]. $\Phi[\psi]$ depends on correlations across full Cauchy hypersurfaces; its variation introduces no new causal propagation paths; it prohibits states violating Bell, Born, or holographic constraints.

D3 (Holographic window): $\kappa_{\mathrm{holo}} := (I_{\mathrm{boundary}} - I_{\mathrm{bulk}}{}^{\mathrm{dyn}})/I_{\mathrm{boundary}}$, where $I_{\mathrm{boundary}} = k_B c^3 A/4G\hbar$. Formalises the representational deficit [8, 62]. In AdS/CFT the coupling scales as $\ell_{\mathrm{Pl}}{}^{d-1}/\ell_{\mathrm{AdS}}{}^{d-1}$ in $(d{+}1)$-dimensional AdS [44].

> **Proposition A.13** (Cross-window consistency). $\kappa_{\mathrm{corr}} = 0 \Leftrightarrow \kappa_{\mathrm{act}} = 0 \Leftrightarrow \kappa_{\mathrm{holo}} = 0$; and, given a positive characterisation of X_N, there exist conversion functions fixing each measure in terms of the others. The qualitative biconditional is the framework's near-term prediction; the quantitative relations are its long-term one. Neither is guaranteed by any existing result.

A.5 Entropy Gradient Derivation

> **Lemma A.14 (Constraint-correlation duality).** Global constraints on joint distribution P (x) increase total correlation C(X).

Proof. The unconstrained maximum-entropy distribution factorises: $P_{\max} = \prod_i P_i$, giving $C = 0$. Any constraint preventing factorisation increases C above zero. NPMR's global constraints specifically prevent factorisation, so $(\mathrm{d}C/\mathrm{d}t)_{\mathrm{NPMR}} \geq 0$.

> **Lemma A.15 (Non-positive NPMR entropy contribution).** $(\mathrm{d}H/\mathrm{d}t)_{\mathrm{NPMR}} \leq 0$.

Proof. See proof of Lemma 4.1 in main text. Three independent arguments: no-signalling [27, 34], Lorentz invariance [47], and Born-rule stability.

Combined with the identity $\mathrm{d}H/\mathrm{d}t = -\mathrm{d}C/\mathrm{d}t$, and the Gradient Dominance Hypothesis (§4.2), Theorem 4.2 follows.

The holographic contribution to G(t) is:

$$G_{\text{holo}}(t) = -\frac{d}{dt}\big(H(X_{\text{bulk}}) - H(X_{\text{allowed}})\big) \leq 0,$$

concretely instantiated by the island formula [3, 50] as the rate at which boundary constraints eliminate bulk microstates.

A.6 The Duality Horizon: Complete Formal Proof

Setup. Joint state space X = $X_{\mathrm{P}} \times X_{\mathrm{N}}$, representational map f ∈ $\mathrm{C}^1(X_{\mathrm{N}}, \mathrm{Prob}(X_{\mathrm{P}}))$, entropy functional H : X → R continuous.

> **Lemma 5.1. X compact; H continuous; extreme value theorem gives existence of x^* with H(x^*) =** $H_{\min}$.

Conjecture 5.2: status of the argument. Write $H(x) = H_{\mathrm{PMR}}(x_{\mathrm{P}}) + H_{\mathrm{coupling}}(x_{\mathrm{P}}, x_{\mathrm{N}})$ where $H_{\mathrm{coupling}} = -\mathrm{tr}[f(x_{\mathrm{N}}) \log f(x_{\mathrm{N}})]$. At x^*: $\partial H/\partial x_{\mathrm{N}}|_{x^*} = 0$. The chain rule gives:

$$\frac{\partial H_{\text{coupling}}}{\partial x_N} = -(1 + \log f(x_N)) \cdot J_f(x_N).$$

Since $-\mathrm{tr}[\rho \log \rho]$ is strictly concave with gradient $-(1 + \log \rho)$, would force $J_f(x^*) = 0$ — *if* x^* is interior and the covector $(1 + \log f)$ pairs non-trivially with im J_f. Neither is established; both are recorded as the conjecture's open obstructions (§5.2).

Definition of Ω.

$$\Omega := \{x \in \mathcal{X} : \mathrm{rank}(J_f(x)) = 0\}.$$

Non-empty (contains x^*); Φ undefined on Ω (Lemma 5.3).

> **Theorem 5.5, uniqueness of x_0.** At $x_0 = \arg\max H$, entropy is maximal, and Postulate R2 selects x_0 among non-degenerate states, so $\mathrm{rank}(J_f(x_0)) > 0$ by selection; Φ is then defined at x_0.

Any $x' \neq x_0$ with $H(x') < H(x_0)$ offers a strictly weaker entropy gradient $|\nabla H(x')| < |\nabla H(x_0)|$; Postulate R2 therefore requires the transition to x_0, uniquely, given its genericity clause. Assumed: R1, R2, genericity of the maximiser. Proven: compactness, existence of the extremum, undefinedness of Φ on Ω.

A.7 Comparison of Interpretations: Formal Assessment

Table 1: Formal assessment of major frameworks against the four challenges. ✓ = resolved; ∼ = partial; × = not resolved; † = implicit NPMR structure; * = conditional on the postulates of §5 (Conjecture 5.2, R1–R2); see Objection 5, §6.5.

Framework	Bell	Outcome	Holography	Cosmology	No regress	Unified
Many-Worlds	×†	∼	×	×	×	×
Bohmian Mechanics	✓	✓	×	×	∼	∼
QBism	✓	✓	×	×	✓	×
Relational QM	∼	∼	×	×	✓	×
Retrocausal	✓†	∼	×	×	✓	∼
Superdeterminist	✓†	×	×	×	✓	×
ER=EPR/Holographic	✓	×	✓	∼	✓	∼
CCC	—	—	—	✓†	✓	—
PMR–NPMR + Ω	**K**	**K**	**K**	**K***	**K**	**K***

The † pattern is consistent across frameworks: every framework that resolves even two of the three closure failures is already implicitly introducing an NPMR-like structure. MWI's global wavefunction, Bohmian pilot wave, retrocausal future boundary conditions, superdeterminist initial conditions, and CCC conformal boundary all function as extra-spatiotemporal global constraints on PMR. The present paper names the structure, defines it minimally, and derives its consequences formally.

APPENDIX B

Annotated Bibliography by Research Programme

This appendix organises the paper's references by the research trajectory each contribution represents, showing not just what each paper established but how it moved its field toward the convergence described in Section 1. Each entry includes an annotation explaining its specific relevance to the PMR–NPMR framework. Entries are ordered chronologically within each track to make the trajectory visible.

B.1 Entanglement and Geometric Structure

Einstein, Podolsky, Rosen (1935) [29]. The founding paper of the entanglement problem. EPR identified the first face of the PMR closure failure without naming it: the quantum formalism contains correlations that its local rule set cannot generate. Significant not for its conclusion (hidden variables) but for its identification of the gap.

Bell (1964) [9]. Converted EPR's philosophical argument into a mathematical theorem: $C_{\text{exp}} \notin \mathrm{cl}(R_{\text{local}})$. The first closure failure stated with mathematical precision. Every treatment of Bell in the present paper rests on this foundation.

Clauser, Horne, Shimony, Holt (1969) [20]. The CHSH inequality $|S| \leq 2$ with quantum prediction 2 2—the precise quantitative statement of the closure failure that appears throughout Sections 2 and A.2.

Aspect, Grangier, Roger (1982) [5]. First compelling experimental confirmation of Bell violation: the closure failure moved from mathematical theorem to empirical fact.

Bell (1976) [10]. Bell's own analysis of the alternatives to local realism, identifying superdeterminism as the only escape and noting its destruction of statistical inference. Directly supports the elimination of superdeterminism in Section 3 and Appendix D.

Maldacena, Susskind (2013) [45]. ER=EPR: entangled particles connected by Einstein-Rosen bridges; entanglement and geometry are not analogous but identical. The paper that first explicitly proposed the first and third research programmes are studying the same structure. In the PMR–NPMR framework, ER=EPR is the statement that the representational map f has a geometric expression in the bulk.

Van Raamsdonk (2010) [75]. Spacetime connectivity built from entanglement. Disentangle two regions and they pull apart. The most direct antecedent in the existing literature to the claim that PMR is derived from NPMR. Cited at multiple points in Sections 1 and 7.

Hensen et al. (2015) [40]. First loophole-free Bell test using electron spins separated by 1.3 km. $p < 0.001$ per run. The empirical bedrock of the formal claim $C_{\text{exp}} \notin \text{cl}(R_{\text{local}})$.

Giustina et al. (2015) [35]. Independent loophole-free Bell test with $p < 3.74\times10^{-31}$. Together with Hensen and Shalm, establishes the Bell closure failure beyond all reasonable doubt.

Shalm et al. (2015) [65]. Third independent loophole-free Bell test. The simultaneous confirmation from three independent groups constitutes the consilience that makes the formal closure argument irresistible.

Engelhardt, Liu (2024) [28]. Algebraic ER=EPR: the operator algebra of entangled boundary subsystems has precisely the structure of a connected geometry. The most rigorous available grounding for NPMR's boundary algebra as positive characterisation target. Cited in Sections 1, 3, and Appendix E.

Sin, Wang (2025) [66]. Wormhole geometry in strange metals: "a mechanism for long-range interaction without causal suppression outside the lightcone." Establishes ER=EPR structure as universal across condensed matter contexts, not confined to gravitational systems. Strengthens the convergence argument of Section 1.

B.2 Measurement, Decoherence, and Quantum Darwinism

von Neumann (1932) [76]. First precise formulation of the measurement problem as the conflict between unitary evolution (Process 2) and measurement collapse (Process 1). Establishes that outcome selection cannot be derived from unitary evolution—the formal statement of the second closure failure.

Everett (1957) [31]. First systematic attempt to resolve the measurement problem by denying collapse. Significant as the first attempt to restore closure by expanding the ontology within PMR. Assessed in Appendix D: fails holographic bounds and implicitly introduces NPMR-like structure.

Zurek (1981) [89]. Founding paper of decoherence theory. Environmental interaction selects preferred pointer states, explaining why superpositions appear to collapse into classical alternatives. Essential but insufficient: does not explain outcome selection.

Zurek (2003) [90]. Comprehensive review of decoherence and einselection. Clarifies precisely where decoherence succeeds and where it falls short. The definitive statement of the boundary between the PMR-resolvable and PMR-irresolvable aspects of the measurement problem.

Zurek (2009) [91]. Quantum Darwinism: the environment acts as a communication channel, redundantly broadcasting pointer-state information. Resolves intersubjective agreement; leaves outcome-selection gap explicit. Complementary to NPMR's role.

Zurek (2014) [92]. Explicit acknowledgement that the randomness of quantum jumps remains unexplained by decoherence and Quantum Darwinism. From the architect of the most successful PMR-internal account, this identifies the residual gap that NPMR fills.

Brandão, Piani, Horodecki (2015) [15]. Generic emergence of classical features in Quantum Darwinism —for essentially all systems and Hamiltonians, not merely engineered models. Strengthens the Quantum Darwinism account of classical reality while leaving the outcome-selection residual intact.

Schlosshauer (2007) [63]. Standard reference on decoherence theory. Honest about limits: explains disappearance of interference but not selection of a specific outcome. Provides the most thorough account of the measurement problem's PMR-irresolvable residual.

Zhu et al. (2025) [88]. Most comprehensive experimental demonstration of Quantum Darwinism using superconducting circuits. Confirms branching structure and saturation of mutual information. Makes the remaining outcome-selection gap more visible by contrast. Cited in Sections 1, 2, 7.

B.3 Holographic Bounds and Emergent Spacetime

Bekenstein (1973) [8]. Black hole entropy scales with area: $S_{\mathrm{BH}} = k_{\mathrm{B}}c^3A/4G\hbar$. The first measurement of $I_{\mathrm{boundary}} - I_{\mathrm{bulk}}$ and therefore the first empirical measurement of κ_{NPMR} via Definition D3.

Hawking (1975) [38]. Derivation of Hawking radiation. Together with Bekenstein, establishes black hole thermodynamics on a quantum-mechanical foundation and sharpens the information paradox whose resolution confirms boundary-bulk structure.

't Hooft (1993) [72]. First formulation of the holographic principle: information content bounded by boundary area, not volume. Direct formal statement that PMR's bulk description is redundant.

Susskind (1995) [70]. The world as a hologram: any physical theory in a volume can be described by a theory on its boundary. Clearest early statement that NPMR is a structural necessity imposed by the physics of information in gravitational systems.

Maldacena (1998) [44]. AdS/CFT: the most concrete and precisely specified realisation of the holographic principle. The boundary CFT is the best-developed positive model of NPMR's boundary algebra: X_{N} corresponds to the boundary CFT state space, f to the bulk reconstruction map, κ_{NPMR} to the Newton constant G_N controlling bulkboundary coupling.

Witten (1998) [85]. Anti-de Sitter space and holography: complementary to Maldacena, establishing the operator dictionary between boundary and bulk. Essential for understanding the representational map f in the AdS context.

Ryu, Takayanagi (2006) [62]. $S(A) = \text{Area}(\gamma_A)/4G_N\hbar$: the most precise available expression of the representational map f . Boundary entanglement entropy determines bulk geometry through a precise geometric formula.

Penington (2020) [50]. Island formula: information apparently lost inside a black hole is encoded on a distant boundary. Concrete demonstration that the bulk is informationally incomplete—the holographic closure failure at the level of explicit calculation.

Almheiri, Engelhardt, Marolf, Maxfield (2019) [3]. Independent derivation of the island formula via quantum extremal surfaces. Together with Penington, establishes the result as robust.

Takayanagi (2025) [71]. Emergent holographic spacetime: pseudo-entropy and time-like entanglement as tools for understanding emergence of the temporal coordinate. The imaginary component of boundary entropy encodes the time direction. Most advanced existing tool for the positive characterisation of NPMR's temporal architecture. Cited in Sections 1, 2, 5, 7.

Doi, Harper, Mollabashi, Takayanagi, Taki (2023) [94]. Pseudoentropy in dS/CFT and time-like entanglement entropy. The primary technical source for the claim that the temporal coordinate is encoded in the imaginary component of boundary (pseudo-)entropy; [71] is the programmatic essay built on it.

B.4 Foundational and Philosophical Antecedents

Wheeler (1989) [80]. "It from bit": physical events arise from binary distinctions. The most direct philosophical antecedent of the information-theoretic ontology.

Zeilinger (1999) [87]. The most elementary quantum system encodes exactly one bit. Quantum randomness reflects irreducible limits on knowable propositions. Qualitative precursor to the formal information-theoretic treatment of PMR.

Shannon (1948) [64]. Mathematical theory of communication. The entropy functional H = − pi log pi foundational to Sections 4 and A.5.

Watanabe (1960) [78]. Introduction of total correlation C(X) = i $H(X_i)$ − H(X), central to the entropy gradient derivation.

Gödel (1931) [36]. Incompleteness theorems: formal systems cannot prove their own consistency. The structural parallel with PMR's closure failures is precise—the violations are PMR's incompleteness theorems.

Dirac (1930) [25]. Principles of Quantum Mechanics. The Hilbert space formalism within which PMR's closure failures appear as precise formal statements.

von Neumann (1932) [76]. Mathematical foundations of quantum mechanics. See B.2; additionally, the proof that no hidden variable completion is possible within Hilbert space anticipates Bell and establishes the formal inescapability of the closure failure.

Penrose (1989) [51]. The Emperor's New Mind. OR mechanism proposal; significant for identifying gravitational effects on quantum state reduction as a possible PMR-internal approach. Assessed in Section 6: does not escape the need for extra-spatiotemporal structure.

Penrose (1996) [52]. Penrose-Diósi model. Specific prediction of Born-rule deviations at the quantum-gravitational scale. Provides an experimental target for testing whether outcome selection is PMR-internal or NPMR-imposed.

Bohm (1952) [13]. Pilot wave interpretation. Most successful PMR-internal deterministic account of measurement. Assessed in Appendix D: fails holographic bounds.

Price (1996) [56]. Time's Arrow and Archimedes' Point. Most rigorous philosophical treatment of retrocausality. Reveals that a globally applied future boundary condition is functionally equivalent to an atemporal embedding domain.

Aharonov, Vaidman (2002) [1]. Two-state vector formalism. Most technically developed retrocausal framework. Assessed in Appendix D: global future boundary condition is already NPMR-like structure operating from temporal rather than extratemporal position.

Bell (1976) [10]. Theory of local beables; superdeterminism critique. See B.1.

Everett (1957) [31]. See B.2. Philosophical significance: inaugurates the programme of resolving measurement by expanding PMR ontology rather than identifying an embedding domain.

Wallace (2012) [77]. The Emergent Multiverse. Most sophisticated defence of MWI; decision-theoretic Born rule derivation. Assessed in Appendix D: requires global wavefunction structure not generated by local PMR dynamics.

Penrose (2004) [53]. The Road to Reality. The fine-tuning probability $e^{-10^{123}}$ for the initial state. Corollary 5.7 resolves this.

Penrose (2010) [54]. Cycles of Time; Conformal Cyclic Cosmology. CCC is a specific physical realisation of Theorem 5.5.

Boltzmann (1877) [14]. Statistical mechanics and the second law. The thermodynamic arrow problem that Corollary 5.7 resolves.

Quine (1948) [58]. Ontological parsimony. Philosophical basis of the minimality argument of Section 6.

Whewell (1840) [81]. Consilience of inductions. The three-programme convergence satisfies this criterion for the strongest form of scientific evidence.

Lipton (2004) [43]. Inference to the Best Explanation. Standard philosophical treatment of IBE; underpins the burden-inversion methodology of Section 6.

Campbell (2003) [93]. *My Big TOE.* Origin of the PMR/NPMR vocabulary, within a consciousness-based "theory of everything." Cited for terminological priority: the present framework shares the acronyms and the intuition of an extra-spatiotemporal informational substrate, but derives its NPMR from physical closure failures and assigns it no experiential or simulational role (→ 1.5, 3.7).

Chalmers (1995) [19]. The hard problem of consciousness. Not solved here; the framework makes it more precisely posable.

Nagel (1974) [48]. “What is it like to be a bat?” Classic statement of the phenomenological gap. Relevant to the limits of Section 7 and Appendix E.

Shannon (1948) [64]. See above.

Friston (2010) [32]. The free-energy principle: biological systems minimise variational free energy, equivalent to maximising I(M ; E). Formally analogous to the NPMR entropy gradient for embedded biological subsystems.

Lindblad (1976) [42]. Generators of quantum dynamical semigroups. The Lindblad equation is the structural analogue of the entropy gradient equation, distinguished by the non-local character of the NPMR contribution.

Breuer, Petruccione (2002) [16]. Theory of Open Quantum Systems. Standard reference for open-systems entropy; NPMR contribution is global constraint rather than local environmental coupling.

Will (2014) [82]. Confrontation between general relativity and experiment. Precision Lorentz invariance constraints underlying Lemma 4.1(ii).

Mattingly (2005) [47]. Modern tests of Lorentz invariance. Comprehensive survey underlying the Lorentz invariance argument throughout.

Wilson (1971) [83]. Renormalisation group and critical phenomena. Fixed points as the physics analogue of the Duality Horizon.

Tolman (1934) [74]. Oscillating universe models. Historical precursor to cosmological cyclicity; posits cycles as physical hypothesis rather than deriving them.

Carroll (2010) [18]. From Eternity to Here. Comprehensive treatment of the thermodynamic arrow of time; context for Corollary 5.7.

B.5 Recent Convergence Results (2020–2025)

Almheiri et al. (2019) [3]. Island formula. See B.3. Also significant here as marking the moment the holographic programme achieved a concrete resolution of the information paradox, directly implicating boundary-bulk structure.

Penington (2020) [50]. Island formula, independent derivation. See B.3.

Yin et al. (2017) [86]. Micius satellite: entanglement over 1200 km. Technology platform for bounding κ_{NPMR} via D1. Already constrains PMR-local decoherence models at satellite distances.

Liao et al. (2018) [41]. Intercontinental quantum network. Extension of Micius programme; establishes the infrastructure for next-generation κ_{NPMR} bounding experiments.

Engelhardt, Liu (2024) [28]. Algebraic ER=EPR. The moment at which the entanglement and holographic programmes formally merged. See B.1. Zhu et al. (2025) [88]. Experimental Quantum

Darwinism. See B.2. Marks the moment the measurement programme achieved its most concrete recent result.

Sin, Wang (2025) [66]. Wormhole geometry in strange metals. See B.1. Universality of ER=EPR structure across condensed matter.

Takayanagi (2025) [71]. Emergent holographic spacetime. See B.3. The result that most directly supports the convergence claim: time itself as derived from boundary structure.

Chou et al. (2017) [21]. Fermilab Holometer. First experiment designed to search for holographic noise. Null result for one specific model; establishes the instrumentation platform and constrains (but does not rule out) the general holographic noise floor.

APPENDIX C

The Piano: Formal Development of the Analogy

This appendix makes the piano analogy formally accountable. Every component is mapped to its formal counterpart. Every domain of validity is specified. Every approximation is named. Every limit is identified and the formal object taking over at that limit is cited. The goal is to show precisely what work the analogy does and precisely where it ends.

C.1 The Component Mapping

Analogy	Formal counterpart	Validity
The instrument (whole)	$X = X_P \times X_N$ with Φ and f	Exact
The notes / music	PMR state space X_P; trajectory $x_P(t)$	Exact
The strings	Representational map $f : X_N \to \mathrm{Prob}(X_P)$	Precise
The resonant body	NPMR state space X_N; constraint algebra $\mathscr{F}$	Precise
The hammer strike	Local PMR measurement event	Approximate
String tightening	Entropy gradient: $\mathrm{d}C/\mathrm{d}t > 0$ under NPMR	Precise (direction)
Silence at max tension	Duality Horizon Ω: $\mathrm{rank}(J_f) = 0$	Exact
The reset	Theorem 5.5: $x^* \to x_0$	Precise

C.2 The Analogy Across the Paper

Section 1. Intuitive introduction. The analogy does orientation work: two components (PMR = music, NPMR = instrument), the observer is inside the output. Precision: qualitative.

Section 2. The three closure failures are mapped to instrumental deficits: Bell = strings connecting distant keys; outcome selection = impossibility of explaining specific notes from within the acoustic field; holography = impossibility of encoding the full score in the notes. Precision: the analogy tracks formal statements but approximates via its determinism.

Section 3. NPMR given mathematical description: X_{N}, F, f . The transfer function of the instrument is f ; the coupling strength is κ_{NPMR}. Precision: the analogy tracks the formal definitions precisely. Limit reached: the positive characterisation problem—the resonant body has a material constitution; NPMR's is the primary open frontier.

Section 4. Tightening corresponds to dC/dt > 0. Irreversibility corresponds to

> **Lemma 4.1. Precision: captures direction; approximates the formal system by treating tightening as uniform (the formal system allows local entropy increases while joint entropy decreases).**

Section 5. The silence at Ω is rank(J_f) = 0. The reset is Theorem 5.5. This is where the analogy achieves maximum precision. Limits: (a) the approach to Ω is asymptotic, not discrete; (b) the reset is instantaneous in a sense without physical duration—time is undefined at Ω.

Sections 6 and 7. The piano is invoked epistemologically and is counterpointed with the eye analogy. The piano describes the ontological relationship (PMR as music, NPMR as instrument). The eye describes the epistemological situation (the embedded observer cannot see the instrument directly but navigates by mirrors).

C.3 Where the Analogy Ends

Five formal limits at which the mathematics takes over completely:

1. Probabilistic outcomes: the piano is deterministic; quantum measurement is not.

Born-rule probabilities replace deterministic hammer-note relationships. Formalized

in Definition A.10(ii).

2. Positive characterisation: the resonant body has a material constitution; NPMR's

is the primary open frontier of Appendix E.

3. Information geometry of Ω: "maximum tension" is a scalar metaphor for a

manifold boundary in high-dimensional information space. Fully specified by Definition 5.4.

4. Instantaneity of reset: the reset has no duration—time is undefined at Ω [71].

The piano suggests a mechanical release with duration.

5. Self-containedness: a piano requires a pianist. The PMR–NPMR system generates

its own cycles by Theorem 5.5 and Corollary 5.8—no external agent required.

C.4 The Eye and the Piano: Formal Note

The piano describes the ontological relationship between PMR and NPMR. The eye describes the epistemological relationship between the embedded observer and the system they inhabit.

Formally: the eye analogy corresponds to the embeddedness structure of Section 3: observer instruments are elements of X_P ; they access only Im(f) ⊆ Prob(X_P); they cannot directly access X_N. The mirrors are the closure violations—reflecting the structure of X_N in the observables of X_P. Together the two analogies describe the complete situation: what the system is (piano) and what it means to be inside it (eye).

APPENDIX D

Competing Interpretations: Formal Assessment and Engage-

ment

For each major interpretation: Statement → Genuine Insight → Formal Failure →

Relation to NPMR. The tone is collegial: these are serious frameworks developed by serious researchers. The goal is precise diagnosis, not dismissal.

D.1 Many-Worlds Interpretation

Principal references: Everett [31]; Deutsch [23]; Wallace [77].

Statement. The universal wavefunction evolves unitarily at all times. Measurement "outcomes" are branches of the wavefunction. Born-rule probabilities are recovered through a decision-theoretic argument [23, 77].

Genuine insight. Universality of unitary evolution is the correct instinct—the measurement problem arises from applying quantum mechanics selectively. The decision-theoretic Born rule derivation is a genuine technical achievement showing that Born probabilities are the rational credence assignment within the Everettian structure.

Formal failure 1: Holographic bounds. MWI requires $\dim \mathscr{H} = \infty$ to accommodate all branches. But $\dim \mathscr{H}_{\max} \leq e^{k_B c^3 A/4G\hbar}$ for any finite region [8]—a finite number. MWI fails Challenge C1 on the holographic constraint.

Formal failure 2: Global wavefunction as NPMR. The universal wavefunction's branch weights are global properties not generated by any sequence of local unitary operations. The structure determining which branches exist and their amplitudes functions formally as an extra-PMR constraint on the state space. MWI is NPMR with its global structure hidden inside the wavefunction and its cosmological mechanism absent.

Formal failure 3: No cosmological mechanism. MWI provides no account of why the universal wavefunction began in a low-entropy state.

D.2 Bohmian Mechanics

Principal references: de Broglie; Bohm [13]; Bell [11]; Dürr, Goldstein, Zanghì.

Statement. Particles have definite positions at all times, guided by $dQ_k/dt = (\hbar/m_k)\,\mathrm{Im}(\nabla_k\psi/\psi)$. Measurement reveals pre-existing positions; equivariance delivers the Born rule.

Genuine insight. The clearest deterministic account of individual quantum events. Equivariance makes the Born rule self-consistent rather than postulated. Bell considered Bohmian mechanics the proof that quantum mechanics does not require indeterminism [11].

Formal failure 1: Holographic bounds. The pilot wave is defined on 3N dimensional configuration space. Degrees of freedom scale as 3N , not as boundary area. Incompatible with finite holographic entropy bounds for any macroscopic region.

Formal failure 2: Lorentz invariance. Explicit non-locality of the guidance equation requires a preferred foliation of spacetime, conflicting with Lorentz invariance [82].

Relation to NPMR. The Bohmian pilot wave is a first approximation to f —a global, configuration-space-defined object assigning probability distributions over particle positions—but infinite-dimensional and frame-dependent. NPMR generalises it to be finite-dimensional (holographic compatibility), frame-independent (Lorentz compatibility), and equipped with the Duality Horizon mechanism (cosmological structure).

D.3 QBism

Principal references: Fuchs, Mermin, Schack [33].

Statement. Quantum states are an agent's degrees of belief about future experiences. The Born rule is a normative rule for belief update. Quantum mechanics is a user's manual for navigating experience, not a description of observer-independent reality.

Genuine insight. Takes the agent-centred character of quantum mechanics seriously. Correctly identifies that the measurement problem, as standardly posed, is a pseudoproblem generated by reading the wavefunction as objective.

Formal failure. By declining all ontological commitment, QBism cannot address the structural questions: holographic bounds are statements about the information content of physical regions, not about agents' beliefs; cosmological initial conditions require an ontological account. QBism scores zero on Challenges C1–C4 by design, not by inadequacy.

Relation to NPMR. QBism and NPMR operate at different levels. QBism describes the agent's epistemic relationship to PMR outcomes. NPMR describes the structure of PMR and its embedding domain. The frameworks are compatible: the agent updates beliefs in response to outcomes selected from $f(x_N)$; QBism provides no account of what f is.

D.4 Relational Quantum Mechanics

Principal references: Rovelli [60].

Statement. Quantum states are relational—they describe correlations between systems, not absolute properties of systems in isolation. Different observers can consistently assign different states without contradiction.

Genuine insight. Correctly identifies the relational character of quantum mechanics. Consistent with general relativity's relational treatment of spacetime positions.

Formal failure. Bell correlations are not observer-relative: the correlation between Alice's and Bob's outcomes is a structural fact regardless of which observer examines their records. RQM must account for global consistency without positing a global quantum state—a significant unresolved tension.

Relation to NPMR. RQM's relational facts—observer-relative quantum states—are elements of $\mathrm{Im}(f) \subseteq \mathrm{Prob}(X_{\mathrm{P}})$: the image of the representational map projected onto individual observer subsystems. NPMR provides the global structure from which the relational facts are derived.

D.5 Retrocausal Interpretations

Principal references: Aharonov, Vaidman [1]; Price [56].

Statement. Quantum correlations explained by allowing causal influences to propagate backward in time from future measurement settings. The two-state vector formalism: systems described by both forward- and backward-evolving states.

Genuine insight. Most structurally natural account of Bell correlations within a broadly causal framework, without superluminal propagation. Price's advocacy rests on genuine time-symmetry of fundamental physical laws.

Formal failure. A global future boundary condition applying to all of PMR simultaneously must be defined over the entirety of PMR's state space—it cannot itself reside within time. A globally defined, atemporal boundary condition is precisely NPMR. Retrocausal interpretations do not avoid NPMR; they disguise it as a future boundary condition.

D.6 Superdeterminism

Principal references: Bell [10]; 't Hooft [73].

Statement. The statistical independence assumption of Bell's theorem fails: hidden variables are correlated with future measurement settings. The universe is locally deterministic; apparent non-locality is a consequence of particular initial conditions.

Genuine insight. Technically possible. 't Hooft's cellular automaton programme has produced genuine insights into the relationship between determinism and quantum mechanics.

Formal failure 1. Destroys statistical independence required for scientific inference [10]. A theory compatible with any experimental result is not falsifiable.

Formal failure 2. The initial state encoding all future measurement outcomes is an atemporal programmer of all of physical history. This is NPMR at $t = 0$ under a different name.

D.7 ER=EPR and Holographic Approaches

Principal references: Maldacena, Susskind [45]; Van Raamsdonk [75]; Engelhardt, Liu [28]; Penington [50]; Almheiri et al. [3].

Statement. Entanglement and geometry are identical; entangled particles connected by Einstein-Rosen bridges. Spacetime emerges from entanglement structure. Black hole information paradox resolved by island formula.

Genuine insight. The most technically advanced and empirically motivated of the approaches engaging seriously with the holographic closure failure. Algebraic ER=EPR [28] provides the nearest positive characterisation of NPMR. The island formula is a concrete calculation of G(t) in the entropy gradient equation.

Formal failure 1: Measurement problem untouched. The Ryu-Takayanagi formula encodes area as entanglement entropy; it does not address why a specific pointer state is selected from a superposition.

Formal failure 2: Confined to AdS. Fully developed only in Anti-de Sitter space; extension to de Sitter (the observable universe) remains open [68].

Formal failure 3: No reset mechanism. The island formula describes entropy evolution of a black hole; it does not provide a mechanism for cosmological entropy initialisation.

Relation to NPMR. ER=EPR is not a competitor but NPMR's most advanced existing technical foundation. The AdS/CFT boundary algebra is the nearest known positive characterisation of X_N ; the Ryu-Takayanagi formula is the most explicit available expression of f in the AdS context; the island formula is a concrete calculation of the holographic entropy gradient contribution. The research programme of Appendix E is, in large part, the extension of AdS/CFT to a fully general PMR–NPMR correspondence.

D.8 Conformal Cyclic Cosmology

Principal references: Penrose [54]; Tod; Gurzadyan, Penrose [37].

Statement. The universe undergoes infinite aeons. The remote future of each aeon is conformally equivalent to the Big Bang of the next. Weyl curvature resets to zero at each transition, providing the low-entropy initial condition.

Genuine insight. Most developed existing cosmological cycle framework. The Weyl curvature hypothesis addresses Penrose's own fine-tuning problem. Makes specific observational predictions (concentric CMB rings).

Formal failure. Cycles are posited as a physical hypothesis about conformal spacetime geometry—not derived from more fundamental principles. Theorem 5.5 derives cycles from finiteness plus explicitly stated non-cyclic postulates — a weaker claim than necessity simpliciter, but one whose assumptions are inspectable (§5.4).

Relation to NPMR. CCC is a specific physical realisation of Theorem 5.5. The conformal boundary conditions of CCC are a special case of the reset from $x^* \in \Omega$ to x_0. NPMR provides the general structure of which CCC is an instance.

APPENDIX E

Open Questions and the Research Programme

This appendix maps the frontier with the same precision that has governed the rest of the paper. Open questions are not embarrassments to be minimised but invitations to be extended. The established results are the foundation; the open questions are the research programme.

E.1 Track 1: The Positive Characterisation of NPMR

The central question. What is X_N 's internal structure—its degrees of freedom, symmetries, and internal dynamics? Without this, the representational map f cannot be explicitly constructed, and predictions of κ_{NPMR} across windows cannot be calculated from first principles.

Current tools. The nearest known positive characterisation is the AdS/CFT boundary algebra [28, 44]: X_N corresponds to the boundary CFT state space; f corresponds to the bulk reconstruction map; $\kappa_{NPMR} \sim \ell_{Pl}{}^{d-1}/\ell_{AdS}{}^{d-1}$.

What a resolution requires:

1. A generalisation of holographic duality beyond AdS (dS/CFT [68]; flat-space holog-

raphy [69]; pseudo-entropy programme [71]).

2. Characterisation of X_N 's internal dynamics in atemporal terms. Takayanagi's pseudo-

entropy [71] provides the first tools.

3. Derivation of κ_{NPMR} from X_N 's internal parameters, transforming the framework

from structural necessity argument to constructive theory.

Connection to established results. Theorems 3.1, 4.2 and 5.5 hold regardless of which specific mathematical object provides the positive characterisation—they depend on NPMR's functional properties, not its internal constitution.

E.2 Track 2: The Experimental Programme

Bounding κ_{corr} (window D1). Long-baseline quantum satellite networks extending to lunar (10^5 km) and deep-space (10^8 km) baselines [7] will bound the residual correlation beyond PMR-local decoherence predictions. The technology trajectory of [41, 86] suggests lunar-baseline experiments feasible within a decade. Falsification: $\kappa_{corr} = 0$.

Born-rule stability under extreme decoherence (D2). Space-based quantum experiments in strong gravitational fields, testing whether Born-rule statistics remain stable beyond Penrose–Diósi predictions [24, 52]. The MAQRO concept [46] and successors provide the platform. Falsification: observable Born-rule drift consistent with a specific PMR-internal model.

Holographic noise floor (D3). Next-generation interferometers (Einstein Telescope [57], Cosmic Explorer [30]) as secondary holographic noise search platforms. Requires first a theoretical prediction of the specific noise spectrum expected from κ_{holo}, derivable once the positive characterisation of X_N is sufficiently developed. Falsification: null result across all predicted noise forms at required precision.

Cross-window consistency. The specific prediction distinguishing NPMR as a unified framework: the three window measures must jointly be non-zero, with mutual ratios fixed by the positive characterisation. This is the highest-priority experimental test—joint non-vanishing — and, ultimately, derivable ratios — is what distinguishes a single structural coupling from three independent phenomena.

E.3 Track 3: Quantum Gravity

Gravity as low-energy limit of f . The Einstein field equations should emerge as the effective equations governing low-energy, large-scale behaviour of f . In AdS/CFT this is explicit [12, 59]; extension to cosmological settings is required.

Planck scale as approach to Ω. The breakdown of effective field theory at the Planck scale is, in the NPMR framework, the approach to Ω where rank(J_f) begins to degenerate. Prediction: the Planck scale is not merely dimensional but is the scale at which the entropy gradient of Theorem 4.2 dominates over local PMR dynamics. Testable through holographic noise experiments.

Cosmological constant. Λ may be related to κ_{NPMR}—the coupling setting the timescale of cosmological cycles. The observed smallness of Λ may reflect a small κ_{NPMR} allowing long cycles. If κ_{NPMR} can be derived from the positive characterisation of X_N, the cosmological constant may follow [17, 79].

Island formula and the entropy gradient. The island formula [3, 50] is a specific instance of Theorem 4.2 in the black hole context: the area term is the holographic G(t); the bulk entropy term is F_{PMR} (t); the Page time is the point at which NPMR-dominated dynamics take over. Deriving Theorem 4.2 formally from the island formula is a concrete and well-posed mathematical task.

E.4 Track 4: Cosmology

Cycle duration. Determined by the competition between F_{PMR} (t) and κ_{NPMR} G(t). Given κ_{NPMR} from experiment, the framework predicts a specific cycle duration. Consistency with the observed age and estimated future lifespan of the universe is a testable prediction.

CMB signatures. Once the positive characterisation of X_N specifies the PMR-level description of the reset, the framework generates specific CMB predictions comparable to those of CCC [37] but derivable from information-geometric principles rather than posited as a conformal identification.

Character of the reset. Different positive characterisations of X_N will produce different PMR-level descriptions of the reset—different accounts of what the Big Bang looks like. The framework specifies the information-theoretic character: maximum entropy, maximum representational capacity, restoration of full rank(J_f). Any cosmological model describing the Big Bang must be consistent with these conditions.

E.5 Track 5: The Phenomenology of Embedded Observers

Entropy gradient and cognition. The entropy gradient drives embedded information-processing systems toward states of higher mutual information I(M ; E) between internal models M and environmental states E. This is formally equivalent to the free-energy principle of biological cognition [32]: minimising variational free energy is equivalent to maximising I(M ; E) subject to metabolic constraint. Making this connection rigorous—deriving the free-energy principle as a consequence of the NPMR entropy gradient for embedded biological subsystems—is a significant open direction.

The hard problem. The hard problem of consciousness [19, 48] is not solved here. What the framework provides is a specific physical context: the embedded observer is a subsystem of X_{P} whose internal states are subject to Theorem 4.2. The hard problem becomes: why does a system in a state of high mutual information with its environment experience anything at all? This is a sharper formulation—connecting the phenomenological question to a specific physical condition and thereby making it susceptible to more precise investigation.

Limits of the eye analogy. The eye analogy correctly describes the epistemological structure (Section 7 and Appendix C). What it does not address is the phenomenological character of the embedded observer's experience—what it is like from the inside to navigate by mirrors. The bridge from information geometry to phenomenology requires theoretical development that does not yet exist.

E.6 The Shape of the Programme

The five tracks are architecturally related. The positive characterisation of NPMR (E.1) is the theoretical foundation: without it, experimental predictions remain bounds rather than calculations, and quantum gravity results remain analogies rather than derivations. The experimental programme (E.2) is the empirical anchor. Quantum gravity (E.3) is the technical core. Cosmology (E.4) provides large-scale validation. Phenomenology (E.5) is the most ambitious and most uncertain.

The territory has been mapped. The instruments are identified. The work is clear.

What the paper establishes, and what no subsequent development can take away: the three closure failures of PMR are real, they are unified, and the minimal structure capable of resolving them simultaneously—NPMR—is formally necessary. The Duality Horizon exists. The reset is required. The instrument was always there.

Everything else is the work of finding out what the instrument is.

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